



**MALAYSIAN JOURNAL OF LEARNING  
AND INSTRUCTION**

<https://e-journal.uum.edu.my/index.php/mjli>

How to cite this article:

Nadya Syifa Utami., Sufyani Prabawanto., Phoong Seuk Yen & Arrolld Rynix Anak Rambee. (2025). The reconstruction of functions using didactic transposition: A focus on the textbooks following the Malaysian and Indonesian mathematics curricula. *Malaysian Journal of Learning and Instruction*, 22(2), 184-209. <https://doi.org/10.32890/mjli2025.22.2.9>

**THE RECONSTRUCTION OF FUNCTIONS USING DIDACTIC TRANSPOSITION:  
A FOCUS ON THE TEXTBOOKS FOLLOWING THE MALAYSIAN AND  
INDONESIAN MATHEMATICS CURRICULA**

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Received:13/10/2023

Revised:16/1/2025

Accepted:15/7/2025

Published:31/7/2025

**ABSTRACT**

**Purpose** – This study is aimed at exploring how the topic of functions was organised into knowledge to be delivered in textbooks based on the Malaysian and Indonesian mathematics curriculum, and how the topic reconstruction in both countries differed.

**Methodology** – A documentary research method was used in this study. Study data were collected through the examination of school mathematics textbooks, specifically Grade 8 textbooks in Malaysia and in Indonesia. The data used was the sequence of tasks presented in the chapter on functions in each country's mathematics textbooks. This study analysed data by identifying the learning objectives of functions in both countries' curricula; categorizing task sequences in textbooks using the didactic co-determinacy framework—from domain level to sector level, theme level, and task level; and examining the similarities and differences on the topic of functions and the categorized task sequences in textbooks following the Malaysian and Indonesian mathematics curricula respectively.

**Findings** – The findings indicated that the presentation of functions in Malaysian and Indonesian textbooks followed a similar sequence, beginning with an introduction to functions, progressing to their representation, and concluding with their application in solving real-world problems. Malaysian and Indonesian textbooks both introduced functions as a subset of relations and demonstrate various representations, including arrow diagrams, ordered pairs, tables, formulas, and graphs. While both curricula shared a common foundation in the set-theoretical definition of functions, they differed in their approach to problem-solving. Malaysian textbooks emphasized the use of function graphs more extensively, whereas Indonesian textbooks frequently utilized function formulas as problem-solving tools.

**Significance** – This comparative study provides a practical application of the didactic transposition process of knowledge organised in the mathematics curriculum of two different countries. The results also contribute to the potential pathway for learning functions by drawing lessons from two different systems of education.

**Keywords:** Functions, didactic transposition, Malaysian curriculum, Indonesian curriculum, comparative study.

## INTRODUCTION

Functions play a central role in both mathematics and practical applications. In mathematics, they are essential for studying curves in geometry, analytical expressions in calculus, and mapping in algebraic analysis (Kleiner, 1989, 1993). In practical applications, functions serve as modelling tools for describing, explaining, and predicting real-world phenomena (Michelsen, 2006). Additionally, their historical roots in natural phenomena emphasise their significance in the formulation of physical laws (Kjeldsen & Lützen, 2015). Given these crucial roles, the concept of functions is a fundamental topic in the school curricula on mathematics (Biehler, 2005; National Council of Teachers of Mathematics [NCTM], 2000), including in Malaysian and Indonesian secondary schools (Ministry of Education Indonesia, 2022; Ministry of Education Malaysia, 2018).

Despite the broad applicability of functions in both mathematics and real-world scenarios, research consistently demonstrates that students encounter significant challenges in comprehending this concept (Denbel, 2015; Muzaffer, 2013). These difficulties often originate from a fundamental misunderstanding of the definition of a function, with many students mistakenly believing that the relationship between the domain and range must be bijective, meaning each element in the domain is paired with exactly one element in the range and vice versa (Borke, 2021; Widada et al., 2020).

Indonesian students, in particular, struggle with grasping the input-output relationship that defines a function (Septyawan & Suryadi, 2019). Furthermore, students often confuse function notation, especially when  $f(x)$  and  $x$  are used to represent domain and range elements, respectively, while  $y$  is sometimes used to denote range elements in graphical representations, leading to difficulties in distinguishing between functions and equations (Bardini et al., 2014; Walde, 2017). The transition from algebraic to graphical representations of functions is another significant challenge, as students

frequently struggle to map pairs of values onto a coordinate system accurately (Eames et al., 2021; Panaoura et al., 2015). Similar issues have been observed among Malaysian students, who face difficulties in interpreting function graphs and understanding the distinction between equations and functions (Abdullah, 2010). Additionally, both Malaysian and Indonesian students find it difficult to contextualize functions within real-world applications, particularly in modelling phenomena (Gholami, 2023; Utami et al., 2023).

Students often experience difficulties with functions because their understanding of the concept is incomplete. This means that their mental images of functions, known as their concept image, may not align with the precise definition of functions (Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). While functions have a rigorous formal definition, Sierpinska (1992) argued that when functions are applied in context, they take on meanings beyond the formal definition. Students can interpret the concept differently depending on the context, such as seeing it as algebraic equations, relationships between coordinates on graphs, or mapping (Eisenberg, 1991; Sierpinska, 1992). Although students may be able to handle the practical aspects of functions, their understanding of the theoretical nature of functions can be inconsistent (Bakar, 1991). To address these issues, curriculum analysis can help assess how functions are taught in schools, which mainly shapes the multifaceted meanings students have towards functions.

Numerous studies have examined how functions are presented in the school curricula. Some curricula predominantly focus on the set-theoretical definition of functions (Akkoç & Tall, 2005; Denbel, 2015). A study focused on textbook comparisons had revealed that the US textbook emphasises the real-life application of functions more than the Korean textbook (Hong & Choi, 2018). Moreover, a curriculum analysis has also stressed the importance of linking functions with physics lessons (Sajka & Rosiek, 2019). Recent research highlighted how the sequencing of function materials in curricula significantly influences students' understanding (Ayalon & Wilkie, 2019; Chinofunga et al., 2022; Jukic Matic et al., 2022).

Although these studies made significant contributions to mathematics learning, there is still a lack of cross-national didactical analysis regarding the meaning of functions. One study by Tesfamicael and Lundeby (2019) conducted a comparative textbook analysis on functions based on praxeology, one of the main constructs in the Anthropological Theory of Didactics (ATD). The ATD is a theory developed by Chevallard (2019) that highlights the organisation of mathematical knowledge as a result of human activity in various institutions, rather than being merely abstract. How this organisation differs in each institution is elucidated in another ATD framework, known as didactic transposition, which was utilised as a tool to conduct the comparative curriculum analysis in this study.

### **Didactic Transposition**

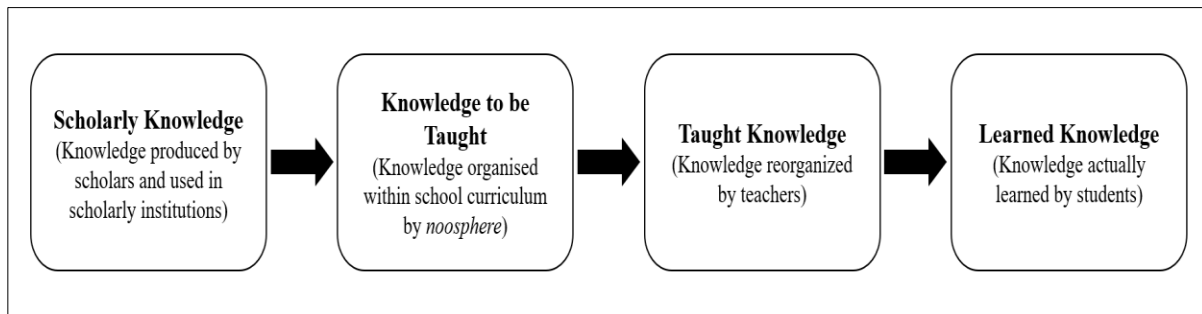
The ATD asserts that cultural conditions where the educational institutions exist impact the diffusion and acquisition of knowledge. This notion stems from Chevallard's idea that different versions of knowledge, denoted as  $k$ , exist and vary depending on their context of use (Chevallard, 2019). In the case of mathematical knowledge, the version found in scholarly institutions differs from the one taught in schools because scholarly knowledge requires a transformation to make it suitable for teaching

(Bosch & Gascón, 2006). However, it is worth examining that this transformation can result in either the preservation of the original meaning or its significant alteration (Kang & Kilpatrick, 1992). To address these challenges of knowledge transformation, Chevallard introduced the theory of didactic transposition.

The didactic transposition points out the transformation of knowledge from its origin in scholarly work, through selection and reorganisation within the curriculum by educational authorities, adaptation for classroom teaching by teachers, until it is ultimately learned by students (Bosch & Gascón, 2014; Lundberg & Kilhamn, 2018). This process typically unfolds in four sequential stages, as illustrated in Figure 1.

**Figure 1**

*Didactic Transposition Process*



Notes. Modified from Winsløw (2011)

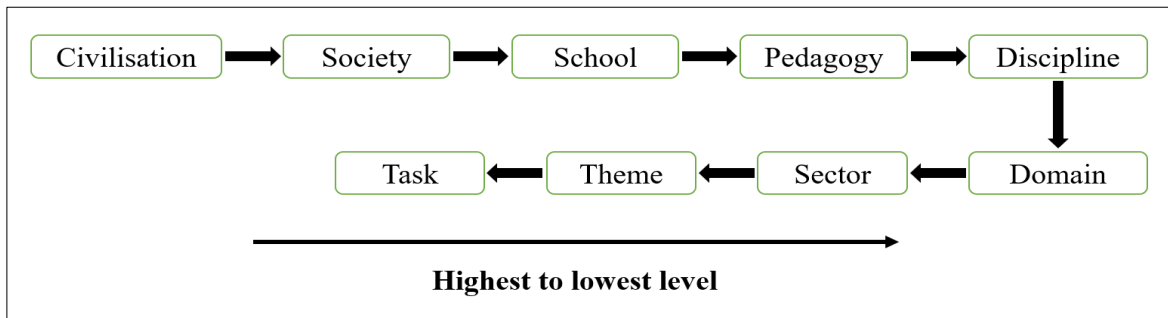
The process of transforming scholarly knowledge into knowledge to be taught, resulting in documents such as the school curriculum, textbooks, and other teaching regulations, is referred to as external didactic transposition (Bosch & Winsløw, 2020; Winsløw, 2011). The subsequent process of reorganising this knowledge is internal didactic transposition, a process adapted to meet the requirements of actual teaching by considering the conditions and constraints of the students. For the purpose of the present study, didactic transposition serves as a theoretical framework to study functions as scholarly knowledge and how the concept is situated into school curriculum in two specific countries: Malaysia and Indonesia. Therefore, this study has provided a deeper analysis of the outcome of the external didactic transposition of both countries.

In didactic transposition, some pieces of knowledge, denoted as  $k$ , is a part of broader knowledge  $K$  and belongs to a discipline  $D$ , which therefore satisfies  $k \subseteq K \subseteq D$  (Chevallard, 2019). Analysing this knowledge  $k$  as an integrated element within  $K$  and  $D$  clarifies its structure, functioning, and distinct uses (Chevallard & Sensevy, 2014). On the other hand, individuals occupying different institutional positions, such as universities, governments, and schools (as depicted in Figure 1), possess a peculiar understanding of mathematical knowledge, shaped by the constraints inherent to their respective positions (Chevallard et al., 2015). Different limitations of knowledge transposition are then explained in the level of didactic co-determinacy (Artigue & Winsløw, 2010) as illustrated in Figure 2. It should be noted that originally, the last level is labelled as *Subject*, which refers to given problems or questions.

However, to avoid any potential confusion with the term *Subject*, typically associated with school subjects (e.g., mathematics), which in this context is referred to as *Discipline*, this study has modified *Subject* to *Task*.

**Figure 2**

*Level of Didactic Co-Determinacy*



Notes. Modified from Artigue and Winsløw (2010)

In this study, using the level of didactic co-determinacy is helpful for studying the position of functions within mathematics, both in Malaysia and Indonesia (from *Domain* to *Task*). Therefore, in comparative studies at the international level, much can be learned from how functions are reconstructed in different educational traditions.

### Reference Epistemological Model (REM) of Functions

Before examining the process of didactic transposition of the knowledge of functions from scholarly knowledge to knowledge to be taught, it is essential to establish a reference model for the knowledge being studied. To this end, ATD introduces the concept of the Reference Epistemological Model (REM). The REM serves as a theoretical framework that outlines the structure of a particular body of knowledge, elaborated by researchers to investigate the process of knowledge transposition in specific contexts (Bosch & Gascón, 2006). In this study, the REM is employed as a guide to analyse how the concept of functions was represented within the mathematics curricula in Malaysia and Indonesia, through a comparative analysis of textbooks from both countries. The following section presents a review of the literature on functions, which was then used in this research as the basis for constructing the REM.

The construction of the REM begins with identifying the concept of functions taught at the middle school level. Tracing the historical evolution of the function concept reveals a shift in its meaning before and after the "new math" era. Prior to this period, functions were primarily understood as a relationship between two quantities that varied simultaneously, such that each value of one quantity determined exactly one value of the other (Kleiner, 1989, 1993). However, following the "new math" era, marked by the introduction of set theory, the concept of functions was generalized to describe a correspondence between two sets, where each element of the first set is associated with exactly one element of the second set (Vinner & Dreyfus, 1989). The earlier conception of functions aids students in understanding the following two key ideas: (1) variables, domain, and range, and (2) functions as processes, especially

in the context of modelling real-world situations (Trujillo et al., 2023). Meanwhile, the later conception presents functions as objects, with sets not restricted to numbers but potentially representing any kind of object. Due to these two differing conceptions, this study has incorporated both in the REM to examine how the function concept is reconstructed in the middle school mathematics curricula of Malaysia and Indonesia.

The second component of the REM emphasizes the representation of functions; specifically, how multiple forms of representation should be systematically introduced to students. Functions can be expressed through tables, formulas, graphs, or sets of ordered pairs, each revealing different aspects of the concept (Ronda, 2015). Consequently, the REM must facilitate a structured learning process for these representations. This study has drawn on Sfard's (1991) dual conception of functions, which distinguished between operational and structural views. The operational conception interprets functions as processes that transform inputs into outputs, often represented by tables or ordered pairs (e.g., arrow diagrams), which help students understand functions as dynamic processes. The structural conception, on the other hand, views functions as objects that describe the overall relationship between inputs and outputs, typically illustrated through graphs. A combination of both views is encapsulated in function formulas, such as  $f(x) = ax$ , which simultaneously describe a process (transforming  $x$  into  $f(x)$  via the rule  $a$  times  $x$ ) and an object (the function  $f(x)$  is consistently defined by  $ax$  across its domain). Therefore, the systematic learning of function representations in the REM begins with tables and/or ordered pairs (operational), progresses to graphs (structural), and ends with formulas, which integrate both operational and structural aspects.

The final component of the REM addresses the application of functions in real-world contexts, extending the use of function representations in problem-solving. As previously discussed, each representation highlights different aspects of a function, making it essential for students to develop the ability to select, adapt, and reason with various representations to solve problems effectively (Duval, 2006; Heinze et al., 2009; Sproesser et al., 2022). By this means, nurturing students' ability to use different function representations appropriately leads to promoting their critical thinking skills, which also effectively applied in problem-based learning in mathematics associated with daily-life (Ismail et al., 2022). In this regard, the study by Rolfes et al. (2018) has categorized real-world applications of functions into the following two types: quantitative and qualitative. The quantitative approach involves numerical computation of the function, such as using tables of values to analyse covariational changes, like rates of change between two quantities. Function formulas is also included in this approach, which is particularly useful when determining specific values, either through direct computation (input to output) or inverse computation (output to input) (Carpenter et al., 2003; Ramful, 2015). The qualitative approach, on the other hand, focuses on understanding the overall behaviour of the function. This is especially relevant when comparing multiple functions or analysing function values within a specific interval. Graphical representations are preferred in this context, as they provide a clear visual illustration of trends and overall changes (Lichti & Roth, 2018; Oliveira et al., 2021).

Following the preceding discussion, this study is aimed at exploring the different ways in which functions were reconstructed within the Malaysian and Indonesian school curriculum. In this study, the term 'reconstructing' refers to the function topic arranged in a curriculum as the result of the selection, organisation, and adaptation process of functions as *a priori* knowledge based on the cultural and

educational constraint of a given country. As the topic of functions encompasses a wide range of concepts and rules taught throughout secondary education, this study has focused on the foundational concept of functions that students first encounter. In both Malaysia and Indonesia, the concept of functions was initially introduced in the 8th grade. Accordingly, the present study has attempted to address the following research questions:

1. How is the didactic transposition of function concept from scholarly knowledge to the knowledge to be taught incorporated into textbooks following the Indonesian and Malaysian Grade 8 Mathematics curriculum?
2. What are the similarities and differences of function using didactic transposition in the Malaysian and Indonesian Grade 8 Mathematics curriculum?

To be able to answer these questions is important as it can aid other researchers, curriculum developers, and practitioners. For researchers, the recent study might provide practical use of didactic transposition as an alternative framework for conducting curriculum comparative studies that would be reproducible by others seeking the same analysis. For curriculum developers, it provides a new perspective of the potential path in learning the function concept for curriculum design. For practitioners, it provides suggestions to teach students the concept of a function and perhaps more broadly in other areas of mathematics.

## **METHODOLOGY**

This study used a documentary research method, which is defined by Bailey (2008) as a method for analysing documents containing information regarding a particular phenomenon. Particularly, this method is used to conduct an investigation and classification of sources, primarily written documents, whether in the private or public domain (Payne & Payne, 2004). In the context of the present study, the method was used to analyse scientific papers related to functions, as well as task sequences in mathematics school textbooks in Malaysia and Indonesia. Subsequently, the results of document analysis described the phenomenon under investigation in the present study, namely the didactic transposition process of function concept from scholarly knowledge to knowledge to be taught in the Malaysian and Indonesian curriculum.

### **Data Collection**

As a general overview, the concept of functions is introduced in lower secondary schools in both countries. In Malaysia, the topic is learned by Form 2 students, while in Indonesia, it is learned by Grade 8 students. Both Grade 8 or Form 2 students are in the same age range (13-14 years old). Table 1 provides an overview of the mathematics topics studied by Form 2 and Grade 8 students as outlined in the mathematics curricula of Malaysia and Indonesia respectively.

**Table 1**

*Mathematics Topics Covered in the Malaysian Form 2 and Indonesian Grade 8 Mathematics Curricula*

Malaysian Curriculum	Indonesian Curriculum
<ul style="list-style-type: none"> <li>• Patterns and Sequence</li> <li>• Factorisation and Algebraic Fractions</li> <li>• Algebraic Formulae</li> <li>• Polygon</li> <li>• Circles</li> <li>• Three-Dimensional Geometrical Shapes</li> <li>• Coordinates</li> <li>• Graphs of Functions</li> <li>• Speed and Acceleration</li> <li>• Gradient of a Straight Line</li> <li>• Isometric Transformations</li> <li>• Measures of Central Tendencies</li> <li>• Simple Probability</li> </ul>	<ul style="list-style-type: none"> <li>• Indices (exponents)</li> <li>• Pythagorean Theorem</li> <li>• Linear Equations and Inequalities of One Variable</li> <li>• Relations and Functions</li> <li>• Line Equations</li> <li>• Statistics</li> </ul>

Based on Table 1, it is evident that the mathematical topics studied by students in Malaysia were more varied compared to those studied by students in Indonesia. Although the Indonesian curriculum covered only six topics, each topic included multiple subtopics. For instance, the topic "Relations and Functions" encompassed subtopics such as sets, Cartesian coordinates, relations, and functions. In contrast, the Malaysian curriculum provided a more detailed list by focusing on individual topics.

Table 1 provides insight into the positioning of functions among other mathematical topics in both countries. In Malaysia, functions were included under the topic "Graph of Functions," which was taught after topics such as "Coordinates" and "Algebraic Formulae." In Indonesia, functions were part of "Relations and Functions" and were taught following the topic "Equations and Inequalities of One Variable." Broadly, there was a similarity in the prerequisite topics for learning functions in both Malaysia and Indonesia, primarily involving algebraic expressions and Cartesian coordinates (included within "Relations and Functions" in Indonesia). This similarity was further reflected in the topics that followed functions, where both curricula introduced line equations (or "gradient of a straight line" in Malaysia) after the study of functions. In conclusion, the positioning of functions in the mathematics curricula of Malaysia and Indonesia was found to be notably similar. Therefore, comparative studies on the teaching of functions in these two countries are worthwhile as it has helped to gain insights into how the concept has been presented, considering their comparable curricular structures.

The primary data for this study were derived from lower secondary mathematics textbooks. In Malaysia, the mathematics curriculum is guided by the Kurikulum Standard Sekolah Menengah (KSSM, or Curriculum Standard for Secondary School), introduced by the Malaysian Ministry of Education in 2017. The selected textbook, *Mathematics Form 2*, by Bahariah et al. (2017), was officially published with government approval, as it has shown alignment with the national curriculum. Similarly, in

Indonesia, the selected textbook aligned with the latest official curriculum issued by the Indonesian Ministry of Education, known as the Merdeka Curriculum. The textbook chosen for analysis was *Matematika SMP/MTs Kelas VIII* (Tohir et al., 2022).

### **Ethical Considerations**

The two mathematics textbooks used in this study were the official textbooks authorized by the Ministry of Education in Malaysia and in Indonesia respectively. This implies that these textbooks are public resources and are freely available as e-books, rather than being published for commercial purposes. For example, in the introduction section of the Indonesian textbook, it is stated (English translation): "*We acknowledge the shortcomings of this book; therefore, feedback and constructive criticism aimed at improving it are highly encouraged from all parties*". Nonetheless, as part of ethical research practices, this study reached out to the respective publishers in both countries to seek permission to use the textbooks, with the aim of advancing and sharing knowledge within the global mathematics education community. Furthermore, the findings of this study have been shared with both publishers to maintain transparency in the use of these textbooks, ensuring mutual benefits for all stakeholders involved.

### **Data Analysis**

The study data were analysed in two phases. In the first phase, the analysis began by identifying the learning objectives of functions in both the Malaysian and Indonesian curricula. This phase was aimed at providing general information about the educational landscape, outlining what each country expected their students to achieve after learning about the concept of functions. Additionally, it shed light on the conditions and constraints influencing the integration of function knowledge within the respective curricular of each country under study.

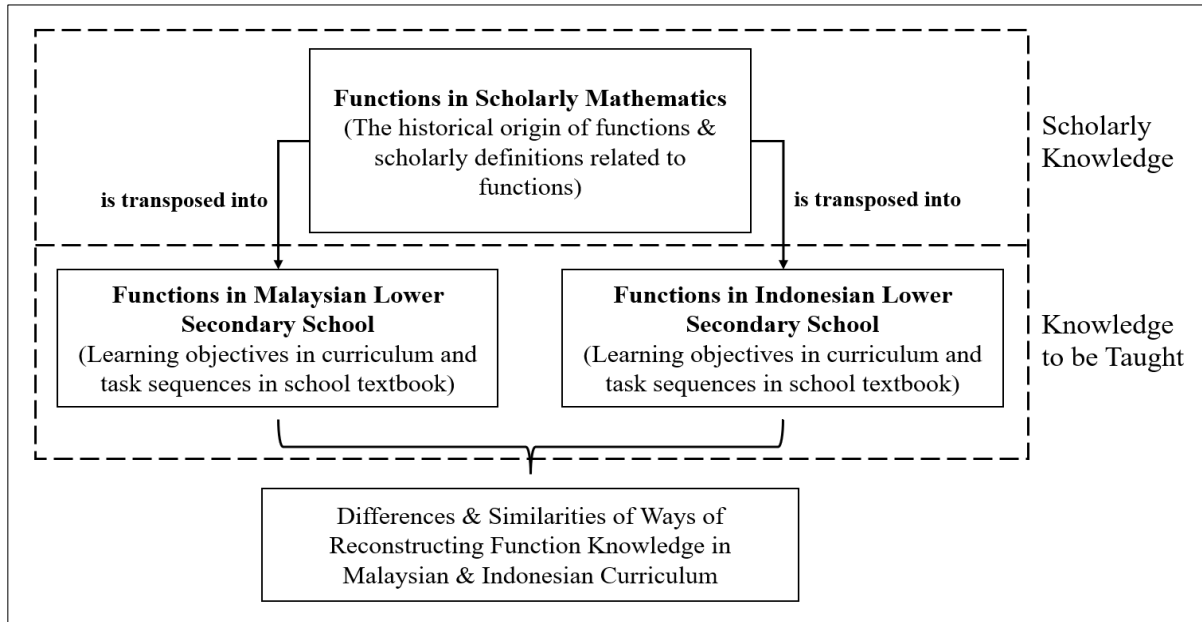
Subsequently, the analysis was conducted on the selected mathematics textbooks for Malaysia and Indonesia, serving as the primary resources that explicitly reflected the learning objectives in the curriculum. To achieve this, an examination of the sequence of tasks and the presentation of function-related concepts within these textbooks were undertaken, employing the level of didactic co-determinacy from Domain to Task level. The Domain level was determined according to the mathematical domain in which the function topic was situated in both countries, namely Algebra. The Sector level was Function, providing discourse on the application of function notions to justify the Theme level. The Theme level was categorised based on the definition and properties related to functions, explaining the Task level. Finally, problems or questions presented in the textbook sharing similar characteristics were grouped into the same task in the Task level. The results of this analysis were then compared with the scholarly knowledge gathered in the first phase to evaluate alignment and discrepancies.

In the final stage of this research, the main aim was to explore the similarities and differences in the reconstruction of the function concept in Malaysia and Indonesia. These investigations were crucial in establishing connections between the two educational systems, giving us insights into how functions were approached and presented in both countries. Additionally, a comprehensive analysis was conducted to determine whether the teaching of functions in these countries aligned effectively with the

scholarly knowledge of functions. This phase was key in highlighting potential gaps that might require adjustments between the official curricula and the academic literature. All data analysis phases are as summarised in Figure 3.

**Figure 3**

*Data Analysis Process of the External Didactic Transposition of Function Concept in Malaysia and Indonesia*



To ensure the trustworthiness of data collection and analysis, this study adhered to the following four key aspects: credibility, objectivity, reliability, and transferability. The most up-to-date mathematics textbooks which aligned with each country’s curriculum were selected to ensure the credibility of data sources, as described in the data collection. To maintain objectivity, the study applied the didactic co-determinacy framework, systematically categorizing the sequence of tasks from the domain level to the task level. Reliability was enhanced through a focused group discussion with experts in mathematics and mathematics education, allowing for cross-verification of task categorization and minimizing researcher bias. For transferability, the study has provided a detailed account of the research setting, objects of analysis, and research context, ensuring that the methodology can be replicated in future studies.

## RESULTS

In this section, the results of the analysis concerning the reconstruction of function concept in the Malaysian and Indonesian curricula through didactic transposition will be presented. The results are presented in accordance to the research questions posed in the study. The first question focuses on the

knowledge to be taught related to functions in each educational setting. Subsequently, the similarities and differences in the presentation of learning sequence of functions in these two countries are presented.

**Didactics Transposition of Function Concept in Textbooks for Indonesian Grade 8 and Malaysian Form 2 Mathematics Curriculum: Knowledge to be Taught**

*Knowledge to be Taught Functions in the Malaysian Curriculum*

To initiate the analysis, Table 2 presents the general information regarding the grade, topic, and learning objectives of functions according to the KSSM of Malaysia. The term "topic" refers to the name of the topic in which function knowledge is organised within the curriculum, while "objectives" outline the expected learning outcomes following the completion of the topic. As illustrated in Table 2, the function topic in Malaysia is organised into two main sections. Firstly, the learning emphasises functions in general, encompassing definitions and multiple forms of function representation. Secondly, the learning delves further into function graphs, including drawing non-linear function graphs and applying function graphs in daily-life situations (e.g., studying trends and making predictions).

**Table 2**

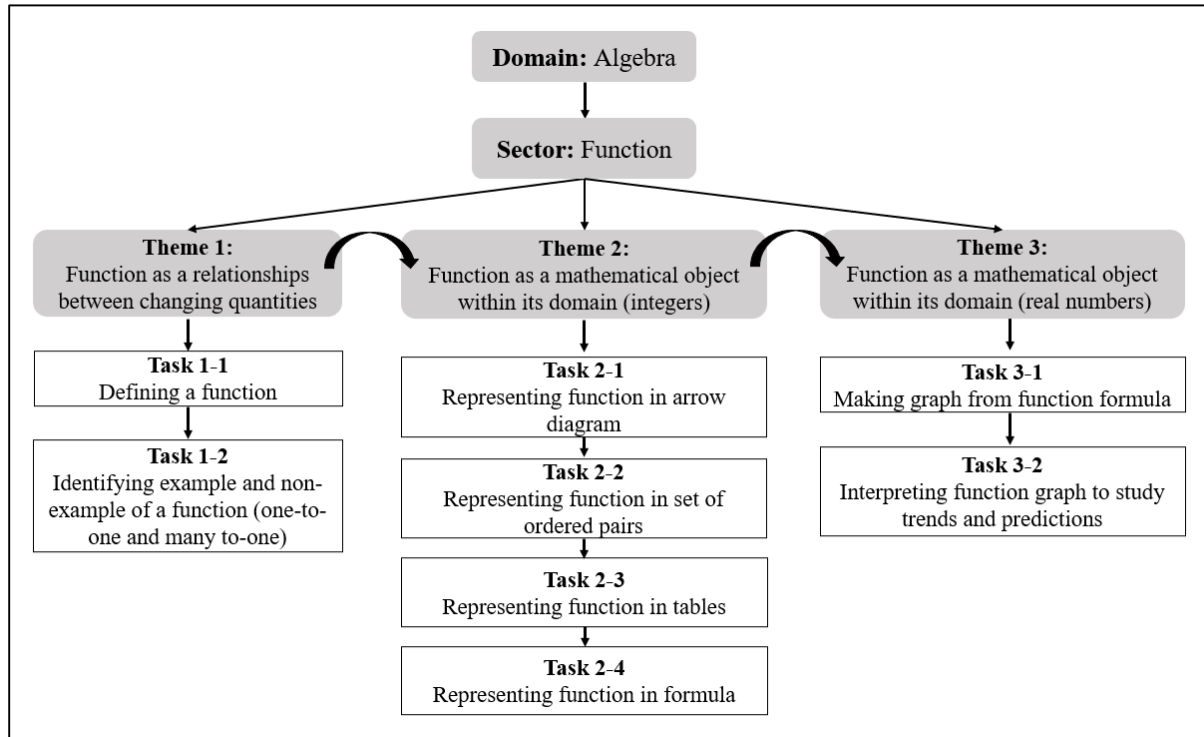
*Learning Objectives of Functions in the Malaysian Mathematics Curriculum*

Grade	Topic	Objectives
Form 2 (Grade 8)	Functions	Explaining the meaning of functions; identifying functions and providing justifications based on function representations in the form of ordered pairs, tables, graphs, and equations
	Graphs of Functions	Constructing tables of values for linear and non-linear functions, drawing the graphs using the scale given; interpreting graphs of functions; solving problems involving graphs of functions.

According to the Malaysian mathematics textbook selected for this study, the function topic was presented under the “Graphs of Functions” chapter. The textbook was further analysed through the organisation of task sequences, which were strategically connected based on the level of didactic co-determinacy, ranging from the domain level to the task level. The resulting representation of the interconnected tasks related to functions in the Malaysian textbook is as presented in Figure 4.

**Figure 4**

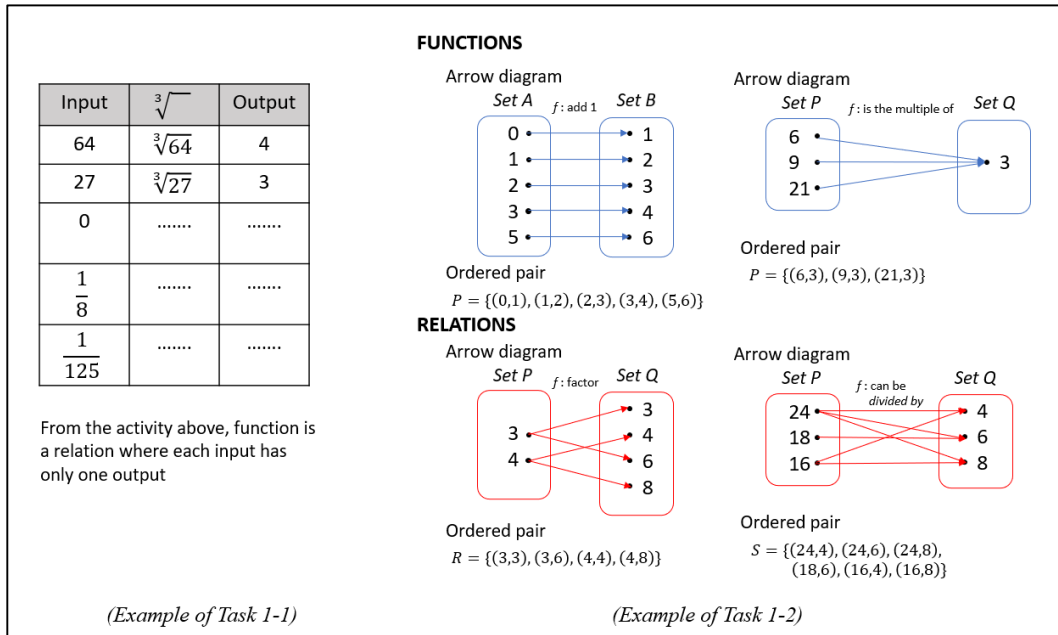
*Function Learning Presented in the Malaysian Mathematics Textbook for Form 2*



As depicted in Figure 4, functions were at the sector level within the domain of algebra. The study of functions encompassed three themes, sequentially presented as Theme 1 through Theme 3. Theme 1 centres around an exploration of function's definition and characteristics. In pursuit of this objective, students were guided through two specific tasks in Task 1-1, determining the output from the given input within tables of values. Concluding this task, the textbook explained that the input was denoted as the domain, while the output was referred to as the range. After defining a function, the sequence progressed to Task 1-2, where the focus shifted to distinguishing between functions and non-functions. Unlike the problem-solving nature of Task 1-2, the two tasks in Task 1-2 served as examples. Ultimately, the completion of Task 1-1 and Task 1-2 was aimed at helping students understand that a function was a relation where each input had only one output, demonstrating the idea of a function as a subset of a relation. Figure 5 below illustrates the examples of Task 1-1 and Task 1-2 as presented in the Malaysian textbook.

**Figure 5**

Examples of Tasks in Theme 1 in a Malaysian Textbook

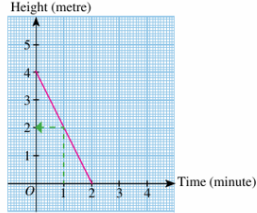


Theme 2 introduced various representations of functions (Task 2-1 to Task 2-4). Even though the textbook defined function as a relation where each input had only one output, reflecting functions as a process or relation between two quantities, the primary forms of function representation introduced by the textbook included arrow diagrams and sets of ordered pairs (Task 2-1 and Task 2-2, respectively). Subsequent tasks involved creating function tables (Task 2-3) and graphs, then formulas (Task 2-4). However, the domain and range sets of functions used in these tasks were subsets of integers, resulting in the function graphs being drawn as points on the Cartesian plane. Problems within Task 2-1 to Task 2-4 shared a similar discourse: a function was a mathematical object. This implies that a function holds the same value across its domain, despite being represented in different forms. Specifically, the domains (and ranges) of functions used were limited to integers.

The final theme, Theme 3, resembled Theme 2 but with increased complexity. While the representations in Theme 3 used functions' domains within integers, Theme 3 provided functions' domains within real numbers. The most notable difference was in graphing the functions. Here, Task 3-1 addressed the creation of the graph of a function formula by listing the coordinate points  $(x, y)$  in tables of values. Figure 6 illustrates the difference between function graphs learned in Theme 2, depicted by points, and function graphs learned in Theme 3, depicted by lines. The exploration of function graphs in Task 3-1 was not limited to linear-function graphs but also included non-linear ones, such as polynomials and exponentials. Following these tasks, Task 3-2 had encouraged students to interpret these graphs effectively. As Malaysian students progressed through this function learning, they were expected to utilise function graphs to study trends and make predictions in daily-life problems.

**Figure 6**

Examples of Tasks in Theme 2 and Theme 3 in a Malaysian Textbook

<p>Given set <math>P = \{1, 2, 3\}</math> and set <math>Q = \{4, 5, 6\}</math>, the function <math>f</math> maps <math>P</math> to <math>Q</math> by adding 3. Represent this function using:</p> <p>(a) Ordered pair      (b) Table (c) Graph              (d) Equation</p> <p>Given set <math>S = \{10, 12, 18, 20\}</math> and set <math>R = \{2, 4, 10, 12\}</math>, set <math>S</math> is mapped to set <math>R</math> by subtracting 8. Represent this function using:</p> <p>(a) Ordered pair      (b) Table (c) Graph              (d) Equation</p> <p style="text-align: center;"><i>(Example of task in Theme 2)</i></p>	<p>(a) Complete the table of values below for the function <math>y = 2x + 4</math>.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;"><math>x</math></td> <td style="padding: 5px;">-2</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">3</td> </tr> <tr> <td style="padding: 5px;"><math>y</math></td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">.....</td> <td style="padding: 5px;">.....</td> <td style="padding: 5px;">6</td> <td style="padding: 5px;">8</td> <td style="padding: 5px;">.....</td> </tr> </table> <p>(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw a graph of the function for values of <math>x</math> from <math>-2</math> to 3.</p> <p>The graph of function shows the movement of a ball that was dropped from a height of 4 metres. Based on the graph, determine</p> <p>(a) The distance of the ball from the ground at the first minute. (b) Time the ball touches the ground.</p> <div style="text-align: right;">  </div> <p style="text-align: center;"><i>(Example of task in Theme 3)</i></p>	$x$	-2	-1	0	1	2	3	$y$	0	.....	.....	6	8	.....
$x$	-2	-1	0	1	2	3									
$y$	0	.....	.....	6	8	.....									

**Knowledge to be Taught Functions in the Indonesian Curriculum**

In the Indonesian mathematics curriculum, learning objectives were not determined according to specific grades (e.g., Grade 8), but rather by phases (*Fase*). The introduction of learning objectives concerning the concept of functions would commence in Fase D, targeting students in lower secondary school spanning from Grade 7 to Grade 9. Table 3 provides a comprehensive overview of the year levels, corresponding topics, and specific learning objectives pertaining to functions as outlined in the Merdeka curriculum of Indonesia.

**Table 3**

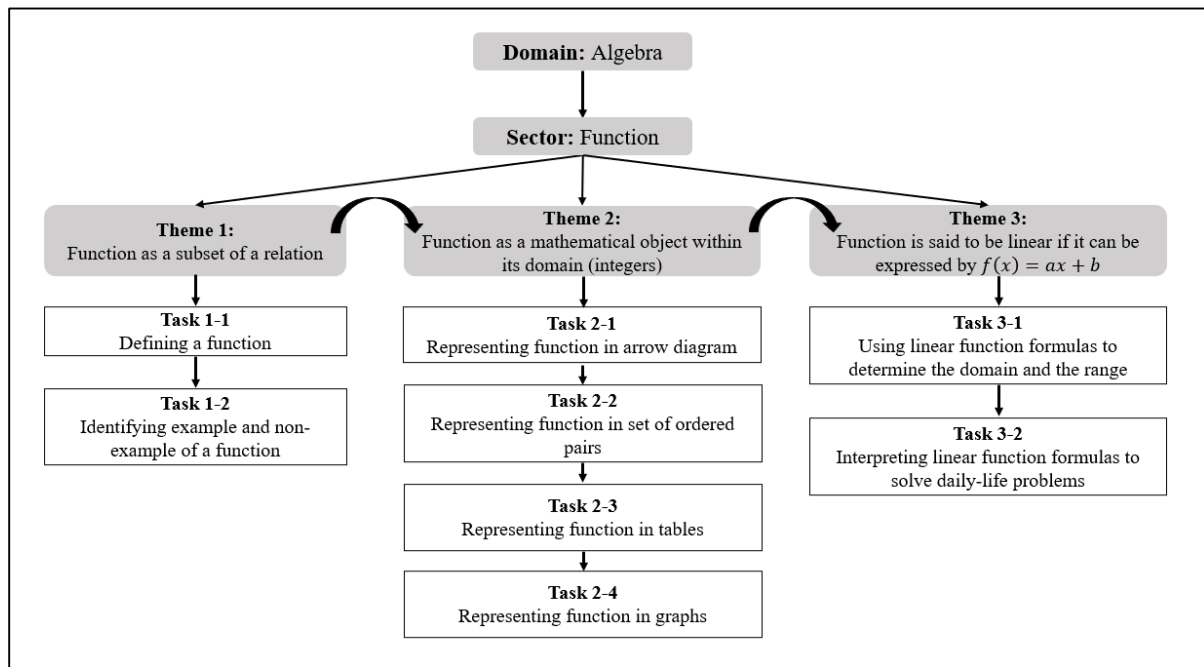
Learning Objectives of Functions in the Indonesian Mathematics Curriculum

Grade	Topic	Objectives
<p><i>Fase D</i> (Grade 7 to Grade 9)</p>	<p>Relations and Functions</p>	<p>Understanding relations and functions (domain, codomain, range); representing relations and functions in arrow diagrams, tables, ordered pairs, formulas, and graphs; representing, analysing, and solving problems using relations and functions.</p>

Although the curriculum did not specify the precise grade level at which the concept of functions should have been introduced, it was initially incorporated into the Grade 8 mathematics textbook. The topic of functions was assigned in the "Relations and Functions" chapter of the textbook. Functions were presented as a sub-topic within this chapter, following the exposition of set theory and relations. For the purposes of this study, the analysis focused solely on the organisation of tasks related to functions, notwithstanding the initial coverage of set theory in the chapter. By employing the concept of didactic co-determinacy, ranging from the domain to the task level, Figure 7 illustrates the interconnectedness of tasks associated with functions in the Indonesian textbook.

**Figure 7**

*Function Learning in the Grade 8 Indonesian Mathematics Textbook*



As illustrated in Figure 7, functions hold a significant position within the domain of algebra, similar to their portrayal in the Malaysian textbook. Within the section on functions, three distinct themes were explored, progressing from Theme 1 to Theme 3. In Theme 1, the objective was to help students understand the definition of a function (Task 1-1), followed by determining whether a given relation constituted a function. Through these tasks, students were expected to grasp the concept that a function represented a unique relation that paired each element in the domain with exactly one element in the codomain, as explained in the textbook. The difference between Task 1-1 and Task 1-2 laid in the context used for functions. While Task 1-1 initially introduced functions using non-formula-based, such as mapping only, Task 1-2 involved both formula-based and non-formula-based as depicted in Figure 8.

**Figure 8**

Examples of Tasks in Theme 1 in an Indonesian Textbook

Filling the table with the password rule given

Kata Asal	Setiap Kata Sandi yang Dihasilkan oleh Aturan Terkait			
	Aturan I	Aturan II	Aturan III	Aturan IV
Komodo	<i>nrprgr</i>			
Harimau		<i>dktykq</i>		
jalak				<i>qzozp</i>
CENDRAWASIH			<i>35e4i1n1j98</i>	
GAJAH		<i>ekbkd</i>		

- Check to see whether the password for each animal is single. Does each animal is coded with only one password?
- Which rule is the safest to use from aturan I to aturan IV? Why do so?

(Example of Task 1-1)

**Example 4.6**  
Let the set  $R = \{3,6,9,12\}$  and the set  $S = \{1,2,3,4,5,6\}$ . "the element of R is three times the element of S" is the relation determined. Is the relation between R and S a function?

**Example 4.7**  
Is the set of ordered pairs from the set B to the set A a function or not a function, if it is known that the set  $B = \{p, q\}$  and the set  $A = \{k, l, m, n\}$ ? Please explain.

1.  $\{(p, k); (q, k)\}$
2.  $\{(p, k); (q, l)\}$
3.  $\{(p, k); (q, m)\}$
4.  $\{(p, l); (q, n)\}$
4.  $\{(p, m); (q, l)\}$
5.  $\{(p, l); (p, m)\}$
6.  $\{(q, n); (q, m); (p, k)\}$
7.  $\{(p, k), (q, l); (p, t)\}$

(Example of Task 1-2)

Following the completion of Theme 1 figure 8, which provided students with a foundational understanding of the concept of functions, the learning progression would then advance to Theme 2. This theme focused on representing functions, emphasising that a function was a mathematical entity whose values remained consistent despite being capable of representation in multiple forms. The initial exposition, defining functions in terms of sets, laid the groundwork for illustrating how this function could be represented. The textbook systematically guided students through various forms of representation in a sequential manner, beginning with an arrow diagram and progressing through ordered pairs, tables of values, graphs, and finally, function formulas  $y = f(x)$ .

While the Malaysian textbook focused on a more comprehensive exploration of function graphs, the Indonesian textbook delved into an in-depth study of function formulas. In the tasks of Theme 3, the Indonesian textbook offered practical examples illustrating the insights gained through an understanding of linear function formulas. Following a structured sequence, commencing with the function expressed as  $y = f(x) = ax + b$ , the textbook addressed issues concerning the procedures for determining the function's value. Furthermore, the textbook presented students with an example demonstrating how to generate a function formula when given two points,  $(x_1, y_1)$  and  $(x_2, y_2)$ . Consequently, Indonesian students were expected to analyse trends and make predictions by engaging with the formulaic or symbolic representation of a function. Figure 9 illustrates examples of Task 3-1 and Task 3-2 as addressed in the textbook.

**Figure 9**

*Examples of Tasks in Theme 3 in an Indonesian Textbook*

<p><i>Example 4.9</i></p> <p>Given that a function <math>f(x) = 3x + 1</math> and <math>f(x) = -2</math>, then determine the value of <math>x</math>.</p>	<p><i>Example 4.10</i></p> <p>Given that a linear function <math>f</math> has value <math>-4</math> when <math>x = -1</math>, and has value <math>5</math> when <math>x = 2</math>. Determine the function formula.</p>
<p><i>(Example of Task 3-1)</i></p>	
<p>There is a reservoir in the house. The water comes to the bathtub from the reservoir through a canal. The height of water in the bathtub reaches 23 liters after 3 minutes, and reaches 47 liters after 7 minutes. the amount of water in the bathtub after being filled for <math>t</math> minutes is given by the formula <math>V(t) = (V_0 + at)</math> liter, where <math>V_0</math> is the volume of water in the bathtub before it is being filled and <math>a</math> is the water rate per minute.</p>	
<p>a. Calculate the volume of the water in the bathtub before filled with water.                  b. How much water is left in the tub after 15 minutes?</p>	
<p><i>(Example of Task 3-2)</i></p>	

**Similarities and Differences of Function Using the Didactic Transposition of Malaysian Form2 and Indonesian Grade 8 Mathematics Curriculum**

Earlier sections discuss how Malaysia and Indonesia adapted and restructured function knowledge to suit their respective educational contexts. Building upon this foundation, this section offers a more detailed exploration of the similarities and differences in the reconstruction of function-related knowledge between these two countries. This analysis has addressed three crucial considerations in function learning within pedagogical practices. Firstly, it examined the function definition that students were expected to comprehend in Malaysia and Indonesia. Secondly, it investigated the sequential order of function representations that each country endeavoured for their students to master. Finally, it identified which aspects of function knowledge each country anticipated their students to utilise when encountering daily-life problems. The similarities and differences in the sequential learning of functions between Malaysia and Indonesia are as outlined in Table 4.

**Table 4**

*Similarities and Differences of Function Learning Between Malaysia and Indonesia*

Category	Similarity	Difference
Function definition	<ul style="list-style-type: none"> <li>● Both countries introduce the concept of function using arrow diagrams (along with sets of ordered pairs) representation</li> </ul>	<ul style="list-style-type: none"> <li>● The Malaysian textbook employs formula-based functions such as "cube of" and "cube root of," whereas the Indonesian textbook employs non-formula-based functions related to "password".</li> </ul>

(continued)

Category	Similarity	Difference
	<ul style="list-style-type: none"> <li>• The function definition is given directly by textbooks after the tasks</li> <li>• Both textbooks give examples of relations that are functions and not functions</li> </ul>	<ul style="list-style-type: none"> <li>• The Malaysian textbook defines function as a relation where each input has only one output; Meanwhile, the Indonesian textbook presented function as a unique relation that pairs each member in the first set to exactly one element in the second set.</li> <li>• The Malaysian textbook uses examples of function and non-function within arrow diagrams, transitioning into graphs and ordered pairs. In contrast, the Indonesian textbook presents examples of functions and non-functions primarily within ordered pairs.</li> </ul>
Function representation	<ul style="list-style-type: none"> <li>• Both textbooks begin with presenting two sets and their relation, then gave practical examples to represent the function (of the two sets given) in multiple ways.</li> </ul>	<ul style="list-style-type: none"> <li>• The representation of functions in the Malaysian textbook commences with arrow diagrams (and sets of ordered pairs), progresses through tables, formulas, and graphs. In contrast, the Indonesian approach initiates with arrow diagrams (and ordered pairs), progresses through tables, graphs, and formulas.</li> </ul>
Function application		<ul style="list-style-type: none"> <li>• The Malaysian textbook provides a more in-depth exploration of function graphs, while the Indonesian textbook primarily emphasises function formulas.</li> <li>• When studying trends and making predictions in daily-life problems, the Malaysian textbook encourages employing function graphs, whereas the Indonesian textbook tends to rely on function formulas.</li> </ul>

As shown in Table 4, both Malaysia and Indonesia introduced the concept of functions to students using arrow diagrams (sets of ordered pairs). Although both countries adopted set representations, the conception of functions learned differed between the two. In Malaysia, the focus was on "the process" of pairing inputs (elements in the first set) with outputs (elements in the second set) according to a given rule, such as  $f$ : *multiple by 2*. In contrast, in Indonesia, the focus was on ensuring the pairing adhered to the characteristics of a function, such as ensuring no element of the first set was paired with more than one element of the second set. For instance,  $\{(A,1), (B,3), (C,7)\}$  is considered a function, whereas  $\{(1, D), (1, G), (2, H)\}$  is not. These differences led to distinct interpretations of the function concept in the two countries. In Malaysian textbooks, the emphasis was on functions as relationships between varying quantities, aligning with the analytic definition of functions. On the other hand, Indonesian textbooks focused on functions as correspondences between sets, reflecting the logical definition of the concept.

After having grasped the concept of a function, the subsequent learning objective involves the representation of functions. Table 4 has highlighted that both Malaysia and Indonesia began their exploration of function representation with arrow diagrams and sets of ordered pairs. In Malaysia, the sequence progressed to include tables of values, formulas, and then formulas, whereas in Indonesia, it proceeded to tables of values, graphs, and finally formulas. Despite the subtle differences in the order of these representations taught to students, this analysis underscores that both the Malaysian and Indonesian curricula had placed a common emphasis on equipping students with the ability to utilise each function representation flexibly. However, it is noteworthy that neither curriculum included tasks that would guide students in applying these representations appropriately to problem contexts.

Finally, a notable difference arises between the Malaysian and Indonesian curricula regarding how they expected students to apply functions in solving real-world problems. Each country was seen as having its own distinct learning objectives. In the Malaysian curriculum, the emphasis was on function graphs, guiding students to construct graphs from tables of values, interpret them, and make predictions based on trends. Conversely, the Indonesian curriculum had focused on function formulas, where students were tasked with interpreting the formula and using it to make predictions in problems. Thus, this finding indicates that the Malaysian curriculum has placed a greater emphasis on function graphs, while the Indonesian curriculum has prioritised function formulas. Despite this distinction, both approaches shared a common objective: enabling students to analyse patterns, relationships, trends, and make predictions in daily-life situations using functions.

## **DISCUSSION**

This study was aimed at exploring the diverse ways in which the concept of functions had been reconstructed within the Malaysian and Indonesian school curricula. The empirical evidence presented in the present study was clearly a practical demonstration of didactic transposition as an alternative, more objective framework for conducting comparative curriculum studies. Furthermore, the findings provided a novel perspective on potential learning trajectories for functions at the lower secondary school level. This paper has organized the discussion of its findings according to the proposed research questions as follows.

In addressing the first research question, the findings shed light on the conceptual interpretations of functions as outlined by Kleiner (1989, 1993) and Ponte (1992) — functions as relationships between quantities (process) and functions as correspondences between sets (object). The study's detailed analysis of the initial entry points in both curricula — Malaysian and Indonesian — has provided concrete examples of these theoretical constructs in practice. The Malaysian textbook's emphasis on input-output processes aligned with the "process" view, emphasizing functions as dynamic relationships between varying quantities. Meanwhile, the Indonesian textbook's focus on non-numerical sets and pairing elements resonated with the "object" view, focusing on the pairing of elements between sets without the process of deriving an output from an input. Furthermore, while both countries' textbooks frequently used sets of ordered pairs (and arrow diagrams) as initial representations of functions, the Malaysian textbook applied rules for pairing elements, whereas the Indonesian textbook did not

emphasize these rules. This empirical validation has helped deepen our understanding of how these theoretical distinctions could manifest in curriculum design.

The study has also contributed to our understanding of Sfard's (1991) dual conception of functions (operational vs. structural) by demonstrating how different curriculum designs aligned with these views in their sequencing of function representations. The Indonesian textbook's progression followed similar conceptions initiated by Sfard (1991), from tables (operational) to graphs (structural) and then formulas (integrating both). On the other hand, the Malaysian textbook, while also logical, diverged slightly by presenting function formulas before graphs. Each approach has its own strengths: The Indonesian curriculum reflected a progression from process to object, while the Malaysian curriculum better prepared students for understanding line equations through its earlier introduction of function formulas and graphs.

In terms of function application, the study has been able to identify a crucial gap in current Malaysian and Indonesian textbook practices. Both textbooks primarily focused on the technical procedure of converting functions into various representations but overlooked the critical aspect of selecting the appropriate function representation for specific problems. Ronda (2015) highlighted that each function representation has its strengths and weaknesses, depending on the problem-solving strategy required (Rolfes et al., 2018; Sproesser et al., 2022). Fostering students' ability to reason among different function representations is crucial for concept formation and cognitive flexibility in problem-solving (Duval, 2006; Heinze et al., 2009). Nonetheless, the tasks within the textbooks from both countries did not address this need. This has a significant practical implication for teaching. This study strongly suggests that lessons should actively require students to engage with the strengths and weaknesses of different function representations (tables, graphs, formulas) and to select the most suitable one for a given problem-solving situation. Fostering this ability is critical for developing cognitive flexibility and problem-solving skills in students, moving beyond mere procedural knowledge.

Addressing the second research question, this study has highlighted the unique strengths of each curriculum's reorganization of function knowledge. The Malaysian emphasis was on interpreting function graphs and their connection to real-life contexts. There is an advantage to the Malaysian approach, as previous studies have shown that graphical representation of functions is often closely related to depicting functional relationships in real-life contexts (Lichti & Roth, 2018; Oliveira et al., 2021). When transforming a situation into a function graph, Sproesser et al. (2022) have suggested that students must understand the underlying relationship between the involved variables, determine the set of points  $(x, y)$ , and identify the pattern so that a graph line can be drawn. Listing the set of points  $(x, y)$  can be facilitated by creating a table of values, which later allows students to establish the corresponding relationship between variables. This generalisation can be used to draw the graph line. Practically, this study has given educators a new perspective to integrate more graph-centric activities on functions that encourage students to translate real-world scenarios into graphical representations.

Whilst the Malaysian curriculum excelled in encouraging a deeper interpretation of function graphs, this however, did not imply that the Indonesian approach, which emphasised function formulas, was less effective. This study has revealed that the Indonesian curriculum had focused on the following three critical tasks related to function formulas: 1) generating the function formula  $f(x)$ , 2) determining  $f(x)$

as the image of  $x$  under  $f$ , and 3) determining  $x$  as an element in the domain of  $f$  such that the image of  $x$  is  $f(x)$ . The first two tasks are common in solving problems related to function formulas. However, the third task promotes students' reversible thinking. As suggested by Ramful (2015), such problems require students to perform reverse operations on the function, laying the groundwork for understanding inverse functions. The ability to perform reverse calculations in functions is essential in its contribution to developing algebraic thinking, particularly in enhancing equation-solving ability (Carpenter et al., 2003). Hence, the Indonesian curriculum offered an advantage by helping students to work on reverse problems in functions. This implies that educators can incorporate more problems that require reverse operations to prepare students for inverse functions and enhance their algebraic proficiency.

Based on the findings discussed above, this study broadly offers significant contributions to both the theory and practice of mathematics education, particularly concerning the teaching and learning of functions.

From a theoretical standpoint, this research has provided empirical evidence supporting the practical use of didactic transposition (Bosch & Gascón, 2006, 2014; Chevallard, 2019; Chevallard & Sensevy, 2014) as a framework for conducting comparative curriculum studies. By applying this framework, the study was able to objectively analyze and articulate the different ways in which the scholarly knowledge of functions was transformed and presented within the Malaysian and Indonesian school curricula. This demonstrates didactic transposition's utility in offering a systematic and objective lens for understanding how mathematical concepts are recontextualized for educational purposes.

In terms of practical implications for teaching and learning mathematics, this study has highlighted the importance of different entry points and conceptual interpretations of functions. Teachers can benefit from their understanding that introducing functions as dynamic relationships (Malaysian approach) or as correspondences between sets (Indonesian approach) could influence students' initial grasp of the concept. Teachers can strategically choose or combine these approaches based on their students' needs and the desired learning outcomes, fostering a more comprehensive understanding of functions. The findings also provided a new perspective on potential pathways in learning about functions at the lower secondary school level. The detailed analysis of the structuring of function representations in both countries offered valuable models for curriculum developers and textbook authors. While both approaches have their strengths, this study has suggested that a deliberate sequencing of representations (e.g., operational to structural, as seen in Indonesia, or preparing for line equations earlier, as in Malaysia) can impact student learning. Teachers can consider these sequences when designing lessons, ensuring a logical progression that supports concept formation.

In sum, the findings of this study can serve as a reference for pedagogical considerations, particularly in designing function learning for lower secondary school students. It encourages educators to move beyond a single, prescribed approach and instead draw upon the strengths of different curricular designs to create richer and more effective learning experiences.

## **CONCLUSION**

Using the didactic transposition theory, this study has explored how Malaysia and Indonesia reconstructed the function knowledge in their respective mathematics curricula for use in textbooks in their respective countries. By focusing on the analysis from scholarly knowledge to knowledge to be taught, this study has obtained two key findings. First, the didactic transposition of functions in two neighbouring countries. In the Malaysian curriculum, the sequential learning of functions consists of the following three themes: function as a subset of a relation, function as a mathematical object within the domain of integers, and function as a mathematical object within the domain of real numbers. In contrast, the Indonesian process aligned with the two themes in Malaysia but chose to interpret its linear function formulas as the last theme. Second, there were different ways of approaching the reconstruction of functions in the two countries. In introducing the function concept, Malaysia started with numerical functions entailing functions through the input-output process, while Indonesia began with non-numerical functions, focusing on pairings the elements of both sets. At the end of function learning, the Malaysian curriculum expected the students to solve problems related to trends and predictions using function graphs, while the Indonesian students were expected to use function formulas.

Building on the insights garnered from this study, it has become imperative for future research to extend this investigation into the subsequent phase of didactic transposition, specifically focusing on internal didactic transposition. This involves examining how the knowledge of functions, once organized in textbooks (knowledge to be taught), is subsequently transformed and presented as taught knowledge within actual classroom lessons.

Such an exploration would be invaluable for several reasons. Firstly, it could illuminate the subtle, yet significant shifts and adaptations that occur as teachers interpret and deliver curriculum content. This includes how pedagogical choices, classroom interactions, and student engagement might further reshape the conceptual understanding of functions. Secondly, by comparing the organization of function knowledge as presented in textbooks with its actual manifestation in lessons, future research can reveal crucial similarities and differences. This comparative analysis would offer deeper insights into the effectiveness of curriculum design and identify potential areas where pedagogical support is most needed.

## **ACKNOWLEDGMENT**

We wish to express our gratitude to the Faculty of Mathematics and Science Education at Universitas Pendidikan Indonesia and the Faculty of Mathematics and Science at Universiti Pendidikan Sultan Idris for their invaluable support during this study. We extend our gratitude to the Directorate of Higher Education, Research, and Technology of Indonesia for their support of this study through the PMDSU research scheme.

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