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EMPIRICAL MODE DECOMPOSITION BASED ON THETA METHOD FOR FORECASTING DAILY STOCK PRICE

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ABSTRACT

Forecasting is a challenging task as time series data exhibit many features that cannot be captured by a single model. Therefore, many researchers have proposed various hybrid models in order to accommodate these features to improve forecasting results. This work proposed a hybrid method between Empirical Mode Decomposition (EMD) and Theta methods by considering better forecasting potentiality. Both EMD and Theta are efficient methods in their own ground of tasks for decomposition and forecasting, respectively. Combining them to obtain a better synergic outcome deserves consideration. EMD decomposed the training data from each of the five Financial Times Stock Exchange 100 Index (FTSE 100 Index) companies' stock price time series data into Intrinsic Mode Functions (IMF) and residue. Then, the Theta method forecasted each decomposed subseries. Considering different forecast horizons, the effectiveness of this hybridisation was evaluated through values of conventional error measures found for test data and forecast data, which were obtained by adding

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forecast results for all component counterparts extracted from the EMD process. This study found that the proposed method produced better forecast accuracy than the other three classic methods and the hybrid EMD-ARIMA models.

Keywords: Forecasting stock price, empirical mode decomposition, intrinsic mode functions, theta method, time series.

INTRODUCTION

The challenging task of time series forecasting is a very active as well as an important research area. In many phenomena, the past, present, and future events are correlated intrinsically with various degrees of randomness. Nevertheless, some events are highly unpredictable, whereas some are relatively easy to predict (Makridakis, 1986). Both capturing data characteristics and fitting data with appropriate methods according to characteristics are the methods for better forecasting. Nowadays, there are many statistical as well as machine learning models for time series forecasting. However, hybrid models are also up-and-coming in many cases, and this work proposes an Empirical Mode Decomposition (EMD)-Theta hybrid model.

Choosing or finding the best model for a particular or similar type of time series data is a challenging but essential consideration. Chatfield (1988) discussed competitive effectiveness encompassing the strengths as well as weaknesses of models and approaches regarding aspects of judgmental, univariate, multivariate, automatic, and non-automatic forecasting with a focus on forecasting competitions. It contributed to draw a significant scenic landscape of the contemporary forecasting research works along with a future indication of better forecasting model finding approaches.

There is still much scope and importance of improving forecasting in the field of econometrics and finance where hybridisation can be of particular consideration. In this article, the potential EMD-Theta hybrid model is presented with an evaluation of five Financial Times Stock Exchange 100 Index (FTSE 100 Index) companies along with a comparison of performance with some other classical methods as well as the hybrid EMD-Autoregressive Integrated Moving Average (ARIMA) method. It was evident that the proposed model performed better than other models. This research work found that EMD-Theta overcame limitations and performed over the demerits of other employed models. One potential demerit or limitation of the EMD-Theta model is that EMD can be inapplicable for some time series suffering from end effect and mode mixing, which can be remedied by considering its improved variants. Therefore, EMD-Theta hybridisation has the robustness of better forecasting, which indicates its potential furtherance in research and applications.

LITERATURE REVIEW

Time series forecasting belongs to many research fields where financial and economic types hold a broad and widespread concern and application. The early model developed in time series forecasting is the Autoregressive (AR) model and Moving Average (MA) model. Then, a more developed combined approach of ARIMA was introduced, which is notably followed by the Box-Jenkins approach (Box & Jenkins, 1970; Cholette, 1982). Later on, more modifications evolved and many other models were developed. ARIMA is a superior version of Autoregressive Moving Average (ARMA), which came out of further works (Wold, 1938; Whittle, 1951) as a suitable method for stationary series. ARMA is a consequential combination of two other methods, AR, introduced by Yule (1927) and MA, a work of Slutzky (1937). The method is better written as ARIMA (*p*,*d*,*q*) where *p*, *d*, and *q* stand for or are related to the number of autoregressive terms, the order of integration, and the number of moving average terms, respectively. The value of *d* is obtained by taking differences of the terms one or more times until the series turns to be stationary.

Then, the series is applied for the ARMA process, which can be generally written as in Equation 1:

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + e_t + \beta_1 e_{t-1} - \beta_2 e_{t-2} - \cdots - \beta_q e_{t-q} \quad (1)$$

where x_i and e_j are past values and past deviations or errors, respectively. Using lag operator L^i that operates as $L^i x_t = x_{t-i}$, Equation (1) can be written as Equation 2:

$$\left(1 - \sum_{i=1}^{p} \alpha_i L^i\right) x_t = \left(1 + \sum_{i=1}^{q} \beta_i L^i\right) e_t \quad (2)$$

Along with obtaining the values of *p*, *d*, and *q*, and the fitted values for the coefficients of all terms for better or appropriate model selection and forecasting, the Box–Jenkins approach is followed through some necessary steps. Although the ARIMA method is a classic approach, it is still being applied in different new research areas like a recent work of Zhao et al. (2019) for resource prediction on Kubernetes, an open-source cluster management software. One of the merits of ARIMA is that it performs quite satisfactorily for non-stationary time series, which can be easily and firmly transformed into stationary. However, its mentionable demerit is that ARIMA tends to fail for highly non-stationary and non-linear time series data, especially with turbulent characteristics and dynamic curvature.

Smoothing methods for better data fitting as well forecasting were developed by the seminal works of Brown (1956), Holt (2004), and Winters (1960). Other mentionable contributions in this model were made by Gardner (1985) and Gardner (2006). Exponentially Weighted Moving Average (EWMA), a smoothing technique for time series data fitting, is a work developed by Brown (1956) that historically originated in the 17th century by Denis Poisson in dealing with his numerical analysis problem related to weighted averaging and exponential windowing. A general EWMA model is represented by Equations 3 and 4:

$$E_2 = x_1 \tag{3}$$

$$E_t = \alpha x_{t-1} + (1-\alpha)E_{t-1}, 0 < \alpha < 1, t \geq 3 \tag{4}$$

where $\alpha, x_i,$ and E_i are respectively smoothing parameter, original sequence terms, and exponentially decreasing sequence terms, which are found by convex combinations with original terms. By recursive use, Equations (3) and 4 can jointly be rewritten as Equation 5:

$$E_t = \alpha \sum_{i=1}^{t-2} (1-\alpha)^{i-1} x_{t-i} + (1-\alpha)^{i-1} E_2, t \geq 2 \tag{5}$$

Equation 5 reveals $d(t) = \alpha(1-\alpha)^t$, which has exponential value decrease property towards distant past values. By adding all t smoothing weight, the related cumulative distribution function can be of the form $C(t) = 1 - (1-\alpha)^t$ For effective or optimal smoothing in the EWMA process, the best value of α is a requirement for least deviation or error of fitted data with original data. The Marquardt procedure and other conventional advanced search approaches, as well as manual tuning, are employed to obtain this value. Relevantly, the EWMA method was further developed for an extension for double and triple smoothing by Holt (1957) and Winters (1960).

The Theta method was developed, introduced, and described along with background mathematics by Assimakopoulos and Nikolopoulos (2000) and Hyndman and Billah (2003). The scholars simplified and suggested different approaches for derivation with the same or similar result found. They claimed that the performance of the Theta method was similar to simple exponential smoothing (SES) with a drift. Some other research contributions involving the Theta method are works of Pagourtzi et al. (2008), Nikolopoulos et al. (2011), and Thomakos and Nikolopoulos (2014). In a work by Petropoulos et al. (2019) of inventory performance considering different forecasting methods and approaches participated in M3-Competition, they showed the performance of the Theta method. The work of Spiliotis et al. (2019) embroiled valuable conceptual insight from the Theta method for decomposition, which was extended to non-linear trend and by modifying and improving to a better hybrid model. They presented promising results with the M3-Competition data.

Papacharalampous et al. (2018) applied automatic forecasting methods to the monthly time series data of temperature as well as precipitation. They compared predictability, where the Theta method was an insignificant position among the other models. A different perspective of the Theta method encompassing application and theoretical concepts were discussed and explained by Nikolopoulos and Thomakos (2019), which was wholly dedicated for the Theta method. Since Theta is one of the winners and significant part of forecasting competitions or M-Competitions, a recent brief historical sketch of Hyndman (2020) contained this model. The Theta method (Assimakopoulos & Nikolopoulos, 2000) is an approach for local curvature modification for time series, where the second difference of a newly derived series is related to the second difference of the original series with a scale factor of change or modification. The name of the method is adopted from the related Greek letter θ (Theta) used in the model equation. A time series$\{x_1, x_2, x_3, \dots, x_n\}$of original data and Theta method-based new time series$\{y_1, y_2, y_3, \dots, y_n\}$are related by the following second-order difference equation as in Equation 6:

$$y_t''(\theta) = \theta . x_t'', \tag{6}$$

where $x_t'' = x_t - 2x_{t-1} + x_{t-2}$. Gradual reduction of local curvatures deflates the time series to zero where there is no curvature. Therefore, smaller values of θ contribute towards a more substantial deflation in the curvature pattern. When the value is zero, it produces a straight line or a linear regression line. If θ=1., there is no change in curvature. The authors showed that this approach of curvature modification does not modify or change the mean of the original series.

As per solving the method of second-order difference equation (Kelley & Peterson, 2001), Equation 6 presented by Hyndman and Billah (2003) is of the form of Equation 7:

$$y_t(\theta) = a_\theta + b_\theta(t-1) + \theta x_t \tag{7}$$

Equation 7 is called Theta-line as per original literature, where $\theta = 0, y_t(0)$ contains a linear trend for the time series. The squared error produces Equation 8:

$$e(a_\theta, b_\theta) = [x_t - y_t(\theta)]^2 = \sum_{t=1}^{n}[(1-\theta)x_t - a_\theta - b_\theta(t-1)]^2 \tag{8}$$

For least square error, by minimisation of Equation 8, Equations 9 and 10 are derived:

$$b_\theta = \frac{6(1-\theta)}{n^2-1}\left(\frac{2}{n}\sum_{t=1}^{n} tx_t - (n+1)\bar{x}\right) \tag{9}$$

$$a_\theta = (1-\theta)\bar{x} - \frac{ba_\theta(n-1)}{2} \tag{10}$$

Averaging both sides of Equation (7) will produce Equation (11):

$$\bar{y}(\theta) = a_\theta + \frac{b_\theta(n-1)}{2} + \theta\bar{x} \tag{11}$$

Putting the value of a from Equation10 into Equation 11, $\bar{y}(\theta) = \bar{x}$. Therefore, curvature change through the Theta method does not change the mean of time series dataset.

It can be and is shown by the authors that, $\frac{1}{2}[y_t(\theta,h) + y_t(2-\theta,h)] = x_t$, since $a_\theta + a_{2-\theta} = 0$ and $b_\theta + b_{2-\theta} = 0$. Considering $\theta = 0$ in this method as per the authors, h- step forecast regarding a time series with n- data is , $x_n(h) = \frac{1}{2}[y_t(0,h) + y_t(2,h)]$, where $y_t(2,h)$is found through linear extrapolation and $y_t(0,h)$ is through simple exponential smoothing. Hyndman and Billah (2003) found the equivalent or same result as Assimakopoulos and Nikolopoulos (2000). One expected merit on behalf of the Theta method is that it can capture local curvature occurring due to a new vibration of underlying time series. A possible demerit of this approach is that it may not embroil global average curvature, which in many cases contributes towards the measuring trend.

The seminal work of Huang et al. (1998) introduced the widely applicable Empirical Mode Decomposition (EMD) method as a contribution to signal processing, which later on was applied in diversified fields of research including economics, finance, meteorology, demography, etc. For important indication and inspiration of EMD in financial time series, the work of Huang et al. (2003) occupied an essential place for general guidelines. For the conceptual understanding and explanation of EMD, Rilling and Flandrin (2008) contributed along with insightful illustrations. IMFs, which are a vital part of EMD or Hilbert-Huang transform in the extended case that was emphasized, focused on the concern of the adaptive approach for data analysis in the work of Wang et al. (2010). EMD, an adaptive decomposition, has a close connection with other theoretical methods, namely Fourier transforms and Hilbert transforms. The EMD process divides a signal or sequence of dataset into some sub-signals or sub-sequences of the original data, and these decomposed components are known as Intrinsic Mode Functions (IMF), whereas the last one is called the residue. The number of these components are at most equal to or less than $log_2(N)$, N being the total quantity of data.

IMFs are produced through the algorithmic process named sifting process, which follows the basic concept of Hilbert transforms. Therefore, the decomposed or components are of the same size and form an orthogonal family of sub-signal datasets virtually. In the EMD sifting process, local mean modal values or empirical modes are found through averaging the upper envelope and the lower envelope, which are cubic splines fitted above and below the original signal. Obtaining mean envelops and subtracting them from the immediate remainder dataset are continued in the sifting process until the process ends by satisfying any of the stopping criteria, namely standard deviation (SD), Tracking of Energy Difference, Threshold Method, and S-Number Criterion. Consequently, IMFs are extracted sequentially by following algorithmic steps.

For any signal, let e_u and e_l be the upper and lower cubic spline envelops, then their mean envelope is obtained from Equation 12:

$$m1 = \frac{e_u + e_l}{2} \tag{12}$$

To present the sifting process, let $x(t)$ be a signal and the mean envelope. The following are steps implemented in the process (using Equations 13 – 16):

(a) Obtaining mean envelope $m1$,the process calculates the current remainder by subtracting $m1$ from $x(t)$:

$$h1 = x(t) - m1 \tag{13}$$

(b) In the second step, current remainder $h1$ is treated as the data, and by applying a similar procedure of upper and lower cubic splines, new mean envelope $m11$ is found from $h1$:

$$h11 = h1 - m11 \tag{14}$$

(c) The process is implemented repeatedly, say, k times, until $h1k$, which is satisfied with stopping criterion (Equation 17):

$$h1k = h1(k-1) - m1k \tag{15}$$

(d) When $h1k$ satisfies the stopping criterion, it is regarded the first IMF component of the original data, which can be denoted by $c1 = h1k$. Then, separate $c1$ from the original data: $x(t) - c1 = r1$. This process is performed repeatedly to extract all possible or say, n IMFs and rj:

$$r1 - c2 = r2, \ldots, rn - 1 - cn = rn \tag{16}$$

The stopping criterion based on SD is formulated using Equation 17:

$$SD_k = \sum_{t=0}^{T} \frac{|h_{k-1}(t) - h_k(t)|^2}{h_{k-1}^2(t)} \tag{17}$$

The sifting process is stopped if SD_k has a value less than a pre-set minimum value.

Time series has a similarity with signal processing encompassing patterns, noise along with non-stationarity and non-linearity in some cases. Huang et al. (1998) applied and explained the use of EMD in financial time series. Later on, many works of literature followed the approach and focused on the application of EMD to analyse and hence forecast time series data in many research areas. Although EMD is a powerful decomposition approach, it has some limitations like end effect and mode mixing for which it may not be applied on every time series, but there are improved EMD variants to overcome these limitations.

Many researchers have recommended the hybridisation of EMD with different combinations of models. One such work is of Wang et al. (2014), where their work was on the EMD-ARIMA combined approach for predicting traffic speed in the short-term forecast horizon. Then, the work of Abadan and Shabri (2014) was aimed for rice price forecasting by EMD-ARIMA hybridisation. Later, the work of Nava et al. (2018) was performed on EMD-Support Vector Regression (SVR) for forecasting financial time series of Standard and Poor 500 Index. Meanwhile, Awajan et al. (2017) worked on EMD-MA to forecast the daily stock market index. Next, Nai et al. (2017) had a focus on the EMD-SARIMA-based model for forecasting air traffic. For short-term speed prediction of vehicle-type specific traffic, Wang et al. (2016) applied a hybrid EMD–ARIMA framework. Recently, Zhong et al. (2020) worked on EMD-ARIMA to predict Service Invention Patents in Agricultural Machinery.

In the EMD-ARIMA hybrid method, all of the extracted IMFs along with residue are forecasted using the ARIMA approach, and all these component forecasts are added to produce the forecast results for the original time series. Both EMD and ARIMA are as efficient as their essence. Therefore, sometimes EMD-ARIMA (presented by the procedural diagram in Figure 1 and Algorithm 1 can be a suitable forecasting hybrid method to gain better accuracy. Nevertheless, when they are not in well-accordance for the intrinsic property of underlying time series, especially in weak stationarity cases of IMFs, their hybridisation can be less satisfactory.

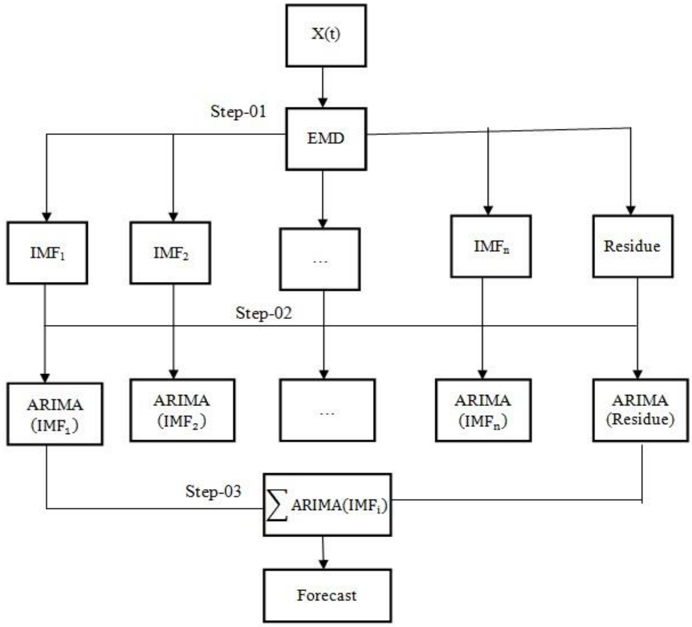

Figure 1. *Procedural diagram of EMD-ARIMA model.*

Algorithm 1: EMD-ARIMA

Input : Selected and pre-processed time series dataset, X
Output : EMD-ARIMA forecast accuracy with forecast accuracies of other methods
Step 1 : **Begin**
Step 2 : Read X
Step 3 : Split X into X_{TRAIN} and X_{TEST}
Step 4 : Set $h \leftarrow |X_{TEST}|$
Step 5 : Compute $X_{forecast.other}$ ←other-method($X_{TRAIN}, n.predict \leftarrow h$)
Step 6 : Define Error(Y, Z)
Step 7 : Compute accuracy.other ←Error($X_{TEST}, X_{forecast.other}$)
Step 8 : Implement EMD(X_{TRAIN})
Step 9 : Store IMFs.X_{TRAIN}, and residue.X_{TRAIN}
Step 10 : **For** $i \leftarrow$ 1 to |IMFs.X_{TRAIN}| **do**
Step 11 : Compute forecast.IMFs[i] ←ARIMA(IMFs.$X_{TRAIN}[i], n.predict \leftarrow h$)
Step 12 : **End for**
Step 13 : Compute forecast.residue←ARIMA(residue.$X_{TRAIN}, n.predict \leftarrow h$)
Step 14 : Compute $X_{forecast.emdARIMA}$ ←add($\sum$ forecast. IMFs[i], forecast.residue)
Step 15 : Compute accuracy.EMD-ARIMA←Error($X_{TEST}, X_{forecast.emdARIMA}$)
Step 16 : Print accuracy.EMD-ARIMA, accuracy.other

THE PROPOSED METHOD

In this work, the = proposed hybrid method was implemented as well as other conventional ones including EMD-ARIMA (Abadan & Shabri, 2014; Wang et al., 2016; Zhong et al., 2020). The hybrid model was performed on daily stock closing price data of Royal Dutch Shell (RDSB), AstraZeneca (AZN), Unilever (ULVR), Reckitt Benckiser Group (RB), and Smith & Nephew (SN) all of which are in the FTSE 100 Index. This study utilises 1,111 data for each of the companies (from January 01, 2015, to April 05, 2019) where the first 1,100 data were separated as training data and the remaining as test data. The 11 test data were also divided into six different forecast horizons of 1, 3, 5, 7, 9, and 11 days. In order to obtain a primary picture of datasets, basic descriptive measures of statistics are presented in Table 1, which encompass mean, median, minimum, maximum, coefficient of variation (COV), skewness, and kurtosis of stock price training data for all the five companies. Since visualisation aids the quick acquisition of data pattern, time series are also presented here in Figure 2.

Table 1

Descriptive statistics

Stock	Mean	Median	Min	Max	COV	Skew	Kurt	Count
AZN	4833.491	4748.5	3774	6525	0.127	0.608	-0.362	1100
RB	6534.615	6538.5	5110	8108	0.096	0.213	-0.811	1100
RDSB	2144.377	2172.5	1277.5	2841	0.153	-0.307	-0.586	1100
SN	1254.123	1241	1051	1545	0.086	0.254	-1.011	1100
ULVR	3600.016	3691.25	2524	4548.5	0.166	-0.161	-1.518	1100

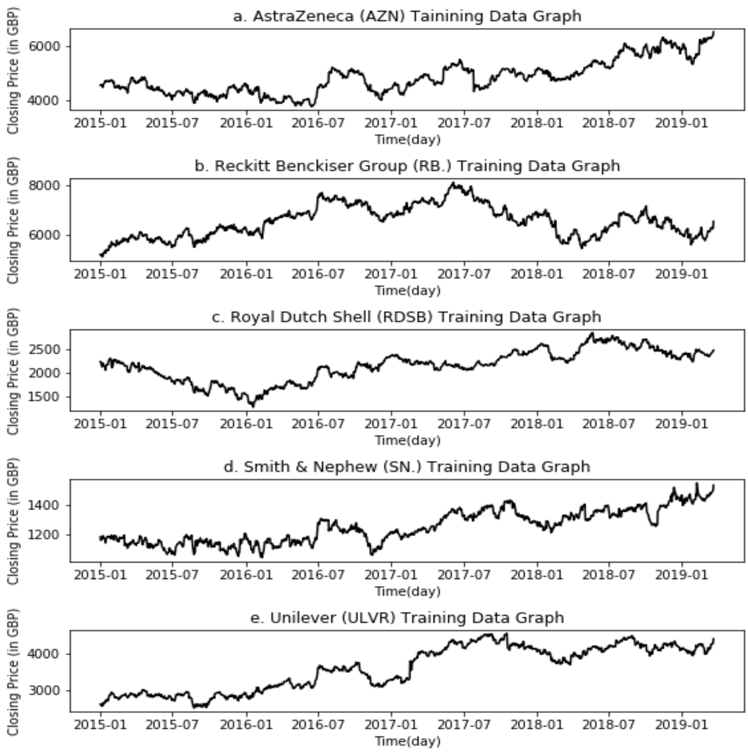

Figure 2. *Graphs of all datasets.*

Proposed EMD-Theta Method

EMD is very useful in dissecting time series data into some nearly orthogonal subseries of different characteristic frequency densities where high-frequency subseries are generally stationary, and low-frequency subseries tend to be non-stationary. Therefore, EMD gives sequential decomposed components. On the other hand, the Theta method has a combined approach of averaging linear trend and simple exponential smoothing following through some Theta values location to location in a view to modify curvature but not mean of the time series data, which can be very useful in some cases. Therefore, hybridisation between EMD and Theta methods, briefly the EMD-Theta model presented in Figure 3 with procedural diagram and Algorithm 2, is a potentially useful approach in time series forecasting. After splitting datasets, this study applied EMD on training sets for IMFs extraction along with the residue and then applied the Theta method on all IMFs as well as the residue for fitting and forecasting to a forecast horizon h . Upon completion of forecast on all component subseries, the final forecast is found by adding these component forecasts. Finally, error measuring or accuracy tools are used to measure the performances that are also compared with forecast results produced by methods of ARIMA, EWMA, Theta, and EMD-ARIMA. Here, the present study worked on six forecast horizons of $h = 1, 3, 5, 7, 9,$ and 11.

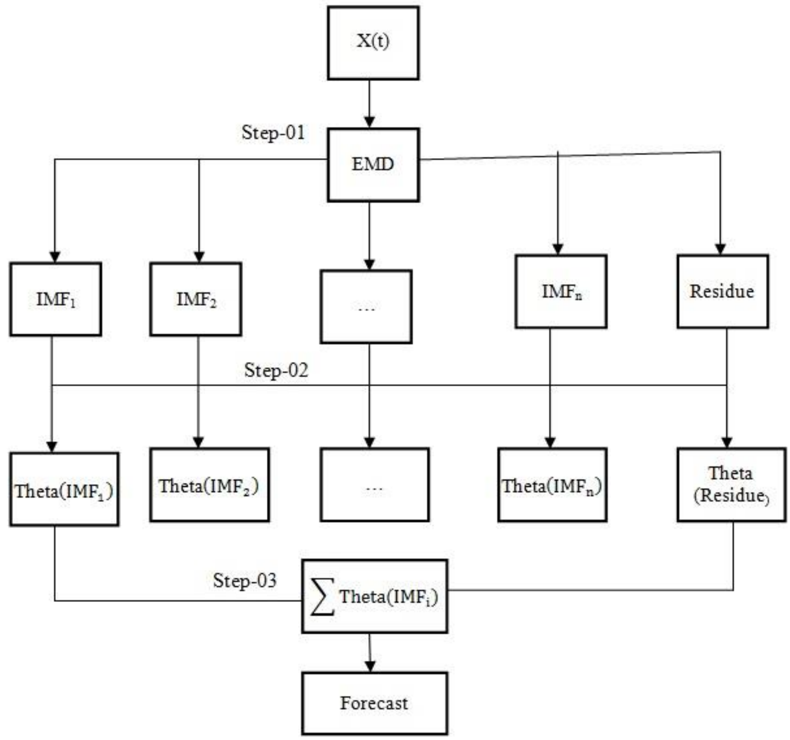

Figure 3. *Procedural diagram of EMD-Theta method.*

Algorithm 2: EMD-Theta		
Input	**:**	Selected and preprocessed time series dataset, *X*
Output	**:**	EMD-Theta forecast accuracy with forecast accuracies of other methods
Step 1	**:**	**Begin**
Step 2	**:**	Read *X*
Step 3	**:**	Split *X* into X_{TRAIN} and X_{TEST}
Step 4	**:**	Set $h \leftarrow \lvert X_{TEST} \rvert$
Step 5	**:**	Compute $X_{forecast.other}$ ←other-method($X_{TRAIN}, n.predict \leftarrow h$)

(continued)

Algorithm 2: EMD-Theta

Step 6 : Define Error(Y, Z)
Step 7 : Compute accuracy.other ←Error($X_{TEST}, X_{forecast.other}$)
Step 8 : Implement EMD(X_{TRAIN})
Step 9 : Store IMFs.X_{TRAIN}, and residue.X_{TRAIN}
Step 10 : **For** i← 1 to |IMFs.X_{TRAIN}| **do**
Step 11 : Compute forecast.IMFs$[i]$ ←Theta(IMFs.$X_{TRAIN}[i], n.predict \leftarrow h$)
Step 12 : **End for**
Step 13 : Compute forecast.residue←Theta(residue.$X_{TRAIN}, n.predict \leftarrow h$)
Step 14 : Compute $X_{forecast.emd\theta}$ ←add($\sum$ forecast. IMFs$[i]$, forecast.residue)
Step 15 : Compute accuracy.EMD-Theta←Error($X_{TEST}, X_{forecast.emd\theta}$)
Step 16 : Print accuracy.EMD-Theta, accuracy.other

For a brief illustration of a typical EMD output, EMD components of AZN training data are shown in Figure 4.

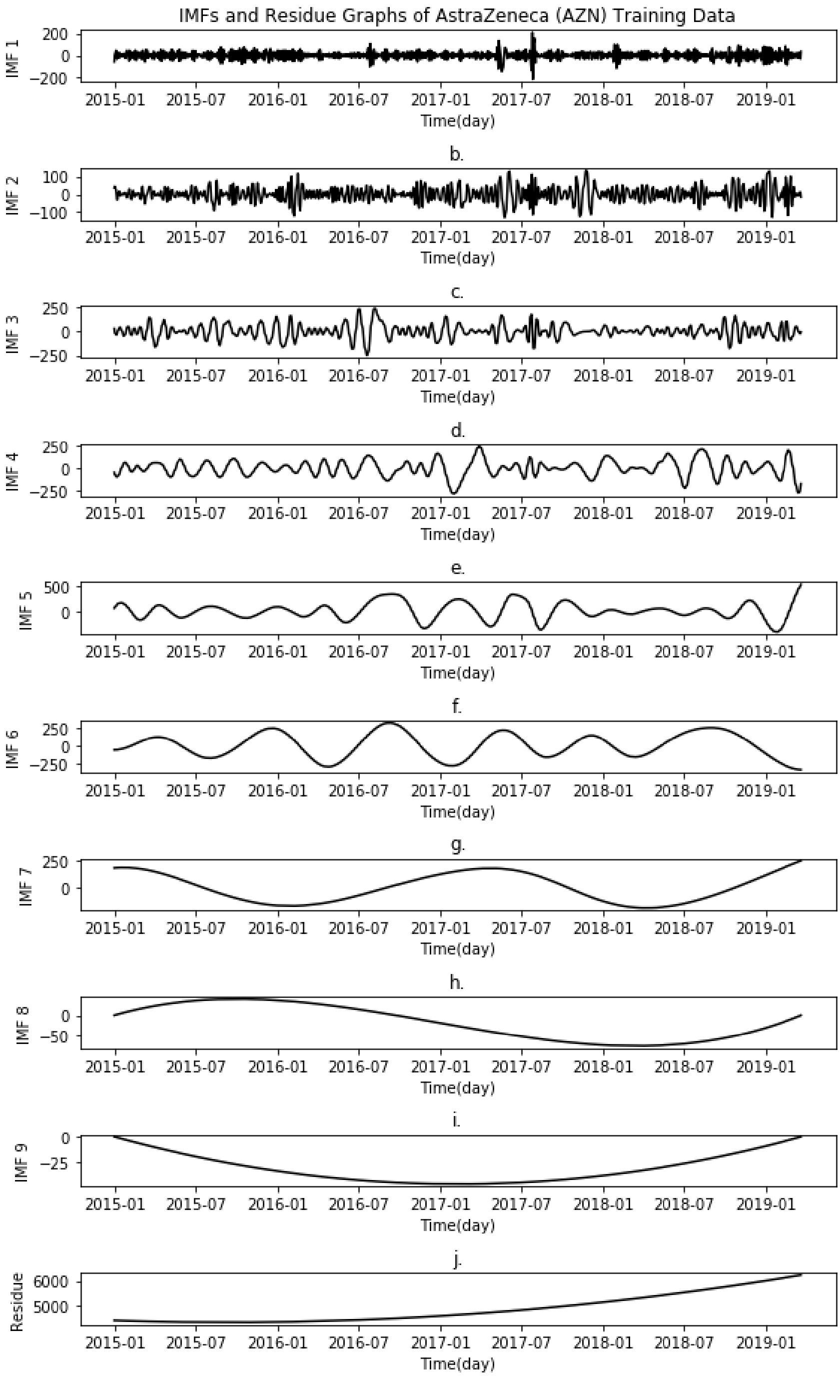

Figure 4. *Illustration of EMD components for AZN.*

Accuracy Measures Used for Forecast Performance Comparison

Better accuracy means less error in conducting forecasting. Although one significant accuracy measuring tool can be essential, other complementary and supplementary tools extend the scope of assurance. Error metrics used in this work are Root Mean Squared Error (RMSE) (Equation 18), Mean Absolute Error (MAE) (Equation 19), Root Mean Squared Relative Error (RMSRE) (Equation 20), Mean Absolute Percentage Error (MAPE) (Equation 21), and Mean Absolute Scaled Error (MASE) (Equation 22) (Hyndman & Koehler, 2006; Shcherbakov et al., 2013; Despotovic et al., 2015). The definitions of this accuracy measures are presented below:

(i) $\text{RMSE} = \sqrt{\frac{\sum_1^n {d_t}^2}{n}}$ (18)

(ii) $\text{MAE} = \frac{\sum_1^n |d_t|}{n}$ (19)

(iii) $\text{RMSRE} = \sqrt{\frac{\sum_1^n (\frac{d_t}{y_t})^2}{n}}$ (20)

(iv) $\text{MAPE} = \frac{\sum_1^n \left|\frac{d_t}{y_t}\right|}{n}$ (21)

(v) $\text{MASE} = \frac{\sum_1^n \left|\frac{d_t}{\frac{1}{n-1}\sum_2^n |y_t - y_{t-1}|}\right|}{n}$, (22)

where $d_t = \widehat{y_t} - y_t, t$ is time sequence and n is total number of data.

RESULTS AND DISCUSSIONS

This section contains the forecasting results (Tables 2–7) of the five time series produced by methods ARIMA (Box & Jenkins, 1970; Cholette, 1982), EWMA (Brown, 1956), Theta (Assimakopoulos & Nikolopoulos, 2000; Hyndman & Billah, 2003; Nikolopoulos & Thomakos, 2019), EMD-ARIMA (Abadan & Shabri, 2014; Wang et al., 2016; Zhong et al., 2020), and the proposed EMD-Theta in six different forecast horizons (h=1, 3, 5, 7, 9, and 11). Error measures RMSE and MAE present comparative performances with absolute errors, while RMSRE, MAPE, and MASE show relative accuracy of methods.

Table 2

Accuracy for forecast horizon, h=1

Dataset	Models	RMSE	MAE	RMSRE	MAPE	MASE
AZN	ARIMA	174	174	0.027	0.027	-----
	EWMA	173.99	173.99	0.027	0.027	-----
	Theta	174.755	174.755	0.028	0.028	-----
	EMD-ARIMA	34.035	34.035	0.005	0.005	-----
	EMD-Theta	**139.573**	**139.573**	**0.022**	**0.022**	-----
RB	ARIMA	130	130	0.02	0.02	-----
	EWMA	127.453	127.453	0.02	0.02	-----
	Theta	127.698	127.698	0.02	0.02	-----
	EMD-ARIMA	323.777	323.777	0.051	0.051	-----
	EMD-Theta	**83.621**	**83.621**	**0.013**	**0.013**	-----
RDSB	ARIMA	66.974	66.974	0.028	0.028	-----
	EWMA	66.998	66.998	0.028	0.028	-----
	Theta	67.381	67.381	0.028	0.028	-----
	EMD-ARIMA	18.194	18.194	0.008	0.008	-----
	EMD-Theta	**64.115**	**64.115**	**0.027**	**0.027**	-----
SN	ARIMA	26.5	26.5	0.018	0.018	-----
	EWMA	25.51	25.51	0.017	0.017	-----
	Theta	25.662	25.662	0.017	0.017	-----
	EMD-ARIMA	52	52	0.035	0.035	-----
	EMD-Theta	**23.467**	**23.467**	**0.016**	**0.016**	-----
ULVR	ARIMA	62.509	62.509	0.014	0.014	-----
	EWMA	62.504	62.504	0.014	0.014	-----
	Theta	63.39	63.39	0.015	0.015	-----
	EMD-ARIMA	58.131	58.131	0.013	0.013	-----
	EMD-Theta	**16.567**	**16.567**	**0.004**	**0.004**	-----

Table 3

Accuracy for forecast horizon, h=3

Dataset	Models	RMSE	MAE	RMSRE	MAPE	MASE
AZN	ARIMA	147.474	145.333	0.023	0.023	4.765
	EWMA	147.465	145.324	0.023	0.023	4.765
	Theta	148.871	146.855	0.023	0.023	4.815
	EMD-ARIMA	142.394	120.577	0.022	0.019	3.953
	EMD-Theta	**114.311**	**111.674**	**0.018**	**0.018**	**3.661**
RB	ARIMA	172.365	167.667	0.027	0.026	2.02
	EWMA	169.889	165.12	0.027	0.026	1.989
	Theta	170.339	165.575	0.027	0.026	1.995
	EMD-ARIMA	463.013	453.692	0.073	0.071	5.466
	EMD-Theta	**127.914**	**121.498**	**0.02**	**0.019**	**1.464**
RDSB	ARIMA	73.728	73.482	0.031	0.031	5.763
	EWMA	74.787	74.498	0.031	0.031	5.843
	Theta	75.564	75.266	0.032	0.031	5.903
	EMD-ARIMA	119.674	91.36	0.05	0.038	7.166
	EMD-Theta	**72.311**	**72**	**0.03**	**0.03**	**5.647**
SN	ARIMA	28.016	27.5	0.019	0.018	2.619
	EWMA	27.045	26.51	0.018	0.018	2.525
	Theta	27.33	26.809	0.018	0.018	2.553
	EMD-ARIMA	39.156	37.636	0.026	0.025	3.584
	EMD-Theta	**25.18**	**24.614**	**0.017**	**0.016**	**2.344**
ULVR	ARIMA	56.385	55.509	0.013	0.013	5.287
	EWMA	56.38	55.504	0.013	0.013	5.286
	Theta	57.981	57.229	0.013	0.013	5.45
	EMD-ARIMA	131.191	120.422	0.03	0.028	11.469
	EMD-Theta	**13.964**	**12.243**	**0.003**	**0.003**	**1.166**

Table 4

Accuracy for forecast horizon, h=5

Dataset	Models	RMSE	MAE	RMSRE	MAPE	MASE
AZN	ARIMA	137.585	126.8	0.022	0.02	1.974
	EWMA	137.576	126.79	0.022	0.02	1.973
	Theta	139.405	129.089	0.022	0.02	2.009
	EMD-ARIMA	263.329	223.321	0.041	0.035	3.476
	EMD-Theta	**107.649**	**94.851**	**0.017**	**0.015**	**1.476**
RB	ARIMA	164.352	161.2	0.026	0.025	3.622
	EWMA	161.855	158.653	0.025	0.025	3.565
	Theta	162.499	159.318	0.026	0.025	3.58
	EMD-ARIMA	512.498	503.733	0.08	0.079	11.32
	EMD-Theta	**119.599**	**115.241**	**0.019**	**0.018**	**2.59**
RDSB	ARIMA	76.469	76.111	0.032	0.032	5.437
	EWMA	77.686	77.298	0.032	0.032	5.521
	Theta	78.854	78.45	0.033	0.033	5.604
	EMD-ARIMA	232.351	188.712	0.097	0.079	13.479
	EMD-Theta	**75.606**	**75.184**	**0.032**	**0.031**	**5.37**
SN	ARIMA	26.687	25.6	0.018	0.017	2.048
	EWMA	25.739	24.61	0.017	0.016	1.969
	Theta	26.135	25.057	0.017	0.017	2.005
	EMD-ARIMA	58.827	53.536	0.039	0.036	4.283
	EMD-Theta	**24.039**	**22.862**	**0.016**	**0.015**	**1.829**
ULVR	ARIMA	48.147	43.905	0.011	0.01	2.281
	EWMA	48.143	43.902	0.011	0.01	2.281
	Theta	49.797	44.769	0.011	0.01	2.326
	EMD-ARIMA	198.58	178.808	0.045	0.041	9.289
	EMD-Theta	**25.28**	**17.778**	**0.006**	**0.004**	**0.924**

Table 5

Accuracy for forecast horizon, h=7

Dataset	Models	RMSE	MAE	RMSRE	MAPE	MASE
AZN	ARIMA	230.987	197.143	0.037	0.032	1.814
	EWMA	230.978	197.133	0.037	0.032	1.814
	Theta	234.048	200.199	0.038	0.032	1.842
	EMD-ARIMA	475.736	384.26	0.077	0.061	3.536
	EMD-Theta	**204.767**	**165.691**	**0.033**	**0.027**	**1.525**
RB	ARIMA	153.072	147.143	0.024	0.023	3.222
	EWMA	150.626	144.596	0.024	0.023	3.166
	Theta	151.403	145.471	0.024	0.023	3.185
	EMD-ARIMA	491.756	481.423	0.077	0.075	10.542
	EMD-Theta	**109.735**	**101.394**	**0.017**	**0.016**	**2.22**
RDSB	ARIMA	68.306	66.166	0.028	0.028	4.436
	EWMA	69.529	67.426	0.029	0.028	4.52
	Theta	70.905	68.964	0.03	0.029	4.623
	EMD-ARIMA	283.972	244.292	0.118	0.101	16.377
	EMD-Theta	**67.733**	**65.697**	**0.028**	**0.027**	**4.404**
SN	ARIMA	22.738	19.714	0.015	0.013	1.955
	EWMA	21.885	18.724	0.015	0.012	1.857
	Theta	22.271	19.318	0.015	0.013	1.916
	EMD-ARIMA	70.635	65.012	0.047	0.043	6.447
	EMD-Theta	**20.396**	**17.123**	**0.014**	**0.011**	**1.698**
ULVR	ARIMA	43.044	37.075	0.01	0.009	1.816
	EWMA	43.04	37.073	0.01	0.009	1.816
	Theta	45.111	38.407	0.01	0.009	1.881
	EMD-ARIMA	236.865	216.983	0.054	0.05	10.628
	EMD-Theta	**27.288**	**19.648**	**0.006**	**0.004**	**0.962**

Table 6

Accuracy for forecast horizon, h=9

Dataset	Models	RMSE	MAE	RMSRE	MAPE	MASE
AZN	ARIMA	251.812	222.222	0.041	0.036	2.096
	EWMA	251.803	222.212	0.041	0.036	2.096
	Theta	255.796	226.046	0.041	0.036	2.133
	EMD-ARIMA	564.326	476.174	0.091	0.076	4.492
	EMD-Theta	**225.308**	**191.388**	**0.036**	**0.031**	**1.806**
RB	ARIMA	150.902	146.111	0.024	0.023	3.238
	EWMA	148.438	143.564	0.023	0.022	3.181
	Theta	149.434	144.649	0.023	0.023	3.206
	EMD-ARIMA	447.398	425.068	0.07	0.067	9.42
	EMD-Theta	**107.341**	**100.572**	**0.017**	**0.016**	**2.229**
RDSB	ARIMA	60.822	55.307	0.025	0.023	3.831
	EWMA	61.985	56.609	0.026	0.024	3.921
	Theta	63.417	58.531	0.026	0.024	4.054
	EMD-ARIMA	280.833	249.219	0.116	0.103	17.262
	EMD-Theta	**60.415**	**55.264**	**0.025**	**0.023**	**3.828**
SN	ARIMA	20.131	16.167	0.013	0.011	2.086
	EWMA	19.345	15.177	0.013	0.01	1.958
	Theta	19.733	15.918	0.013	0.011	2.054
	EMD-ARIMA	70.546	66.158	0.046	0.044	8.537
	EMD-Theta	**18.009**	**13.723**	**0.012**	**0.009**	**1.771**
ULVR	ARIMA	38.045	29.781	0.009	0.007	1.41
	EWMA	38.041	29.779	0.009	0.007	1.41
	Theta	40.147	32.189	0.009	0.007	1.524
	EMD-ARIMA	262.614	243.78	0.06	0.056	11.54
	EMD-Theta	**29.637**	**23.37**	**0.007**	**0.005**	**1.106**

Table 7

Accuracy for forecast horizon, h=11

Dataset	Models	RMSE	MAE	RMSRE	MAPE	MASE
AZN	ARIMA	282.96	253.364	0.046	0.041	2.748
	EWMA	282.952	253.354	0.046	0.041	2.748
	Theta	287.892	257.955	0.047	0.042	2.798
	EMD-ARIMA	646.209	558.56	0.105	0.09	6.058
	EMD-Theta	**256.844**	**223.201**	**0.042**	**0.036**	**2.421**
RB	ARIMA	147.643	143.091	0.023	0.022	3.091
	EWMA	145.177	140.544	0.023	0.022	3.036
	Theta	146.371	141.838	0.023	0.022	3.063
	EMD-ARIMA	404.896	352.454	0.063	0.055	7.612
	EMD-Theta	**104.228**	**97.761**	**0.016**	**0.015**	**2.111**
RDSB	ARIMA	55.688	48.751	0.023	0.02	3.166
	EWMA	56.701	49.816	0.024	0.021	3.235
	Theta	57.936	51.354	0.024	0.021	3.335
	EMD-ARIMA	257.654	221.377	0.106	0.091	14.375
	EMD-Theta	**55.275**	**48.681**	**0.023**	**0.02**	**3.161**
SN	ARIMA	20.715	17.318	0.014	0.012	1.776
	EWMA	19.895	16.328	0.013	0.011	1.675
	Theta	20.509	17.217	0.014	0.011	1.766
	EMD-ARIMA	67.116	62.932	0.044	0.042	6.455
	EMD-Theta	**18.705**	**15.022**	**0.012**	**0.01**	**1.541**
ULVR	ARIMA	34.618	25.955	0.008	0.006	1.392
	EWMA	34.615	25.955	0.008	0.006	1.392
	Theta	36.316	26.488	0.008	0.006	1.42
	EMD-ARIMA	277.411	260.547	0.063	0.059	13.97
	EMD-Theta	**33.401**	**27.615**	**0.008**	**0.006**	**1.481**

It is noticeable from Table 3 until Table 7 that for the five forecast horizons h=3, 5, 7, 9, and 11, the proposed EMD-Theta performed better than the ARIMA, EWMA, Theta, and EMD-ARIMA hybrid methods. This is true by considering relative error measures RMSRE, MAPE, and MASE in the cases of all five FTSE 100 Index companies closing price time series data. Although EMD-ARIMA performed best in case of h=1 (Table 2) for a few companies, it did not play a satisfactory role in other forecast horizons. Overall, the EMD-Theta method performed best of all other approaches.

Intrinsic characteristics and external factor are always responsible for the shape of the data. Provided no uncertainty or external factors are involved, a method can produce relatively better forecast only when the method can capture all the intrinsic features of the dataset and can forecast or extrapolate to future horizons accordingly. The ULVR dataset (with MAPE 0.004, 0.003, 0.004,0.004, 0.005, and 0.008, respectively for h=1, 3, 5, 7, 9, and 11) was best forecastable and RDSB (with MAPE 0.027, 0.03, 0.031,0.027, 0.023, and 0.02, respectively for h=1, 3, 5, 7, 9, and 11) was least forecastable by the proposed method as well as all other methods except the EMD-ARIMA method for h=1. EMD-ARIMA performed best of all other methods in the forecast horizon h=1 with the least MAPE in case of only two companies, i.e. AZN (0.005) and RB (0.008). In h=1, EMD-ARIMA also forecasted better than other methods for the company ULVR (0.015), but not better than EMD-Theta, in which MAPE for ULVR was 0.013.

As per the proposed method, for forecast horizon $h = 1h = 1$, the forecastability of the method based on MAPE could be put according to order for the five datasets from best to worst (with from less to much error values) as ULVR<RB< SN<AZN<RDSB, where the starting one (ULVR) of the sequence was the best case, and the ending one (RDSB) was the worst case among them. As per values of MAPE, the forecastability order for other forecast horizons h=3, 5, 7, 9, and 11 were ULVR < SN<AZN< RB <RDSB, ULVR < SN<AZN< RB <RDSB, ULVR < SN < RB<RDSB<AZN, ULVR < SN < RB <RDSB<AZN, and ULVR < SN < RB <RDSB<AZN, respectively. The overall result tends to indicate that EMD-ARIMA can be the right choice for only single point immediate forecast horizon h=1. However, beyond the single point forecast, EMD-Theta is best of all other models considering the relative accuracy measures.

CONCLUSION

With all the results and discussion in this study, it is evident that the proposed EMD-Theta hybrid method performed better considering all the five non-linear and non-stationary time series datasets and six different forecast horizons based on five types of error or accuracy measures. The ULVR dataset was best forecastable, and RDSB was least forecastable. Future uncertainty and involvement of external factors are primarily responsible for poor forecast results in time series, mainly from short term high-frequency data. Therefore, different data bear different messages of their varying degree of forecastability and some of these phenomena are shown through the selected time series datasets. Nevertheless, for all the five time series datasets, the synergised performance of the EMD and Theta method was better than other methods as capturing, fitting, and forecasting were useful for such data pattern and characteristics. In future, it is expected for further works of forecasting to be extended to machine learning as well as deep learning tools. It is suggested for future studies to focus on unexplored classical model-based hybridisation.

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