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MODIFIED S-CURVE MEMBERSHIP FUNCTION AND ITS APPLICATION TO FUZZY LINEAR PROGRAMMING

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ABSTRACT

The modern trend in industrial application problem deserves modeling of all relevant vague or fuzzy information involved in a real decision making problem. In this paper, the modified S-curve membership function and its methodology in solving real life problems are discussed. In the second part of the paper, the computation of fuzzy linear programming approach is made. Lastly, the usefulness of the modified S-curve membership function is established using a real life industrial production planning of a chocolate manufacturing unit. The unit produces 8 products using 8 raw materials, mixed in various proportions by 9 different processes under 29 constraints. A solution to this problem is achieved, thus establishing the usefulness of the suggested membership function for decision making in maximizing objective function. From the theory and numerical results, it can be seen that the method presented here for solving a system of fuzzy mix product selection problem with modified s-curve membership function is very promising.

Keywords: fuzzy linear programming, satisfactory solution, decision maker, modified S-Curve, vagueness.

1.0 INTRODUCTION

In this paper, a new modified membership function, named as "modified S-curve membership function" is proposed. The modified S-curve membership function is first constructed and its flexibility in taking up vagueness in parameters is established by an analytical approach. This membership function is tested for its useful performance through an illustrative example by employing fuzzy linear programming (Kuz'min, 1981; Carlsson and Korhonen, 1986; Watada, 1997 and Pandian, 2002). The usefulness of this modified S-curve membership function is further established using a real life industrial production planning of a chocolate manufacturing unit. The unit produces 8 products using 8 raw materials, mixed in various proportions by 9 different processes under 29 constraints. This real life problem has been solved and their results are tabulated. The decision maker can also suggest to the analyst some possible and practicable changes in fuzzy intervals for improving the maximum profit.

A general mathematical programming problem involves maximizing or minimizing an objective function subject to various constraints posed by the restrictions within the system. This usually requires that the individual who formulates the mathematical model, such as the analyst and the decision maker or the implementer be very precise in every bit of information even though the system itself may be rather imprecise or vague in nature. However, in real-world decision making problems, the goal and the constraints may be fuzzy in a practical environment. For instance, a decision maker would like the goal to reach some level of degree of satisfaction of the objective function, and allow some tolerance on the constraints instead of actually maximizing the objective function and strictly satisfying the constraints. In these situations, the decision maker can model these problems as fuzzy mathematical programs (Bellman and Zadeh, 1970; Zimmermann, 1991 and Sengupta *et al.*, 2001).

In this paper, the new methodology of fuzzy mix product selection and their application to decision making are studied; especially, fuzzy linear programming based on a vagueness in the fuzzy variables given by a decision maker is attempted.

Various types of membership functions which express a vague aspiration level of a decision maker are proposed, such as a linear membership function (Zimmermann, 1976 and 1978), a tangent type of a membership function (Liberling, 1981), an interval linear membership function (Hannan, 1981), an exponential membership function (Carlsson and Korhonen, 1986) and (Sakawa, 1983), concave piecewise linear membership function (Hannan, 1981) and (Inuguchi et al., 1990), general piecewise linear membership function (Hu and Fang, 1999), logistic membership function (Watada, 1997), an inverse tangent membership function and so on. As a tangent type of a membership function, an exponential membership function, and inverse tangent membership function are a non-linear function. A fuzzy mathematical programming problem defined with a non-linear membership function results in a non-linear programming. Usually a linear membership function is employed in order to avoid non-linearity. Nevertheless, there are some difficulties in selecting the solution of a problem written in a linear membership function. The solution for linear membership function is degenerated (Watada, 1997), therefore the solution cannot be decided uniquely. In order to solve the issue of degeneration, a modified s-curve membership function is employed in this paper to overcome such deficits which a linear membership function has. The first part of the following section is about the statement of problem of chocolate manufacturing, and section 3 is the methodology for constructing a modified s-curve membership function for fuzzy linear programming. Section 4 and section 5 discuss the usefulness of the modified s-curve membership function in obtaining profit maximization through a fuzzy linear programming approach. Lastly, some conclusions and future research work in this area are highlighted.

2.0 PROBLEM OF CHOCOLATE MANUFACTURING

Due to limitations in resources for manufacturing a product, and the need to satisfy certain conditions in manufacturing and demand, a problem of fuzziness occurs in industrial fuzzy systems. This problem occurs also in chocolate manufacturing when deciding a mixed selection of raw materials to produce the varieties of products. This is referred here to as the Product- mix Selection Problem (Tabucanon, 1996).The non-fuzzy data for this problem are taken from the data-bank of Chocoman Inc, USA (Tabucanon, 1996).Chocoman produces a variety of chocolate bars, candies and wafers using a number of raw materials and processes. The objective is to use the modified S-function for obtaining a profit maximization procedure through fuzzy linear programming (FLP).

The objective of the company is to maximize its profit, which is alternatively, equivalent to maximizing the gross contribution to the company in terms of US dollars. The goal is to find the optimal product mix under uncertain technical, raw material and market constraints. Furthermore, it is possible to show the relationship between the optimal profits and the corresponding membership grades (Zimmermann, 1985). According to this relationship, the decision maker can then obtain his optimal solution with a trade-off under a pre-determined allowable imprecision.

3.0 THE METHODOLOGY AND APPROACH

Many problems in science and engineering have been considered from the optimization point of view. As the environment is much influenced by the disturbance of social and economic situations, an optimization approach is not always the best. It is because, under such turbulence, many problems are ill-structured. Therefore, a satisfactory approach may be better than that of optimization. Here, we discuss how to deal with decision problems that are described by fuzzy linear programming (FLP) models and formulated with elements of imprecision and uncertainty. More precisely, we will study FLP models in which the parameters are partially known with some degree of precision.

In this paper, the non-linear membership function is explained in detailed to show how it is used in solving FLP. A form of S-shaped membership function is also proposed. S-curve membership function is proved to be a flexible membership function through an analytical approach. This membership function is to be used in FLP involving fuzzy technical coefficients and fuzzy resource variables.

3.1 Construction of Modified S-Curve Membership Function

The S-curve membership function is a particular case of the logistic function with specific values of B, C and α. These values can be worked out. This logistic function as given by equation (1) and depicted in Figure 1 is indicated as an S-shaped membership function by Zadeh (1971) and Goguen (1969).

$$f(x) = \frac{B}{1 + Ce^{\alpha x}} \tag{1}$$

where B and C are scalar constants and α, $0 < \alpha < \infty$ is a fuzzy parameter which measures the degree of vagueness, wherein $\alpha = 0$ indicates crisp. Fuzziness becomes highest when $\alpha \to \infty$. Notation α determine the shape of membership functions $\mu(x)$, where $\alpha > 0$. It is necessary that parameter α, which determine the figures of membership functions, should be heuristically and experientially decided by experts.

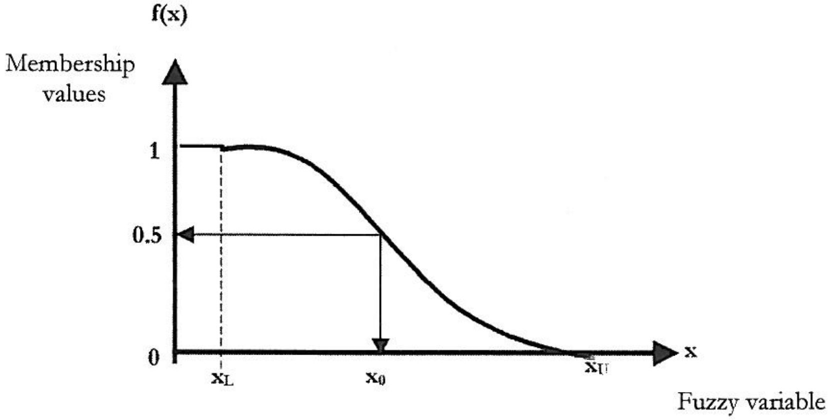

Fig 1: Logistic membership function

The parameter x is considered to be a member of the related fuzzy set ; x_L and x_U are respectively the lower boundary and upper boundary for the fuzzy parameter x. B and C are constants and the parameter $\alpha > 0$ determines the shapes of membership function. The larger the value of α , the more is its vagueness. Figure 1 shows the shape of suggested logistic membership function.

We define, here, a modified S-curve membership function as follows:

$$\mu(x) = \begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \dfrac{B}{1 + Ce^{\alpha x}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases} \qquad (2)$$

where μ is the degree of membership function.

Figure 2 shows the modified S-curve. In Equation (2) the membership function is redefined as $0.001 \leq \mu(x) \leq 0.999$. This range is selected because in a manufacturing system the work force need not be always 100% of the requirement. At the same time the work force will not be 0%. Therefore, there is a range between x^a and x^b with $0.001 \leq \mu(x) \leq 0.999$. This concept of range of $\mu(x)$ is used in this paper.

It should be noted that a linear membership function shows a necessity level and a sufficiency level at their grades 0 and 1 respectively, On the other hand, for a non-linear membership function, such a modified s-curve function has a necessity level and a sufficiency level which may be approximated at the points with grades 0.001 and 0.999, respectively (Watada, 1997).

We rescale the x axis as $x^a = 0$ and $x^b = 1$ in order to find the values of B, C and α. Novakowska (1977) has performed such a rescaling in his work in the social sciences.

The values of B, C and α are obtained from equation (2) as

$$B = 0.999\,(1 + C) \qquad (3)$$

$$\frac{B}{1 + Ce^{\alpha}} = 0.001 \qquad (4)$$

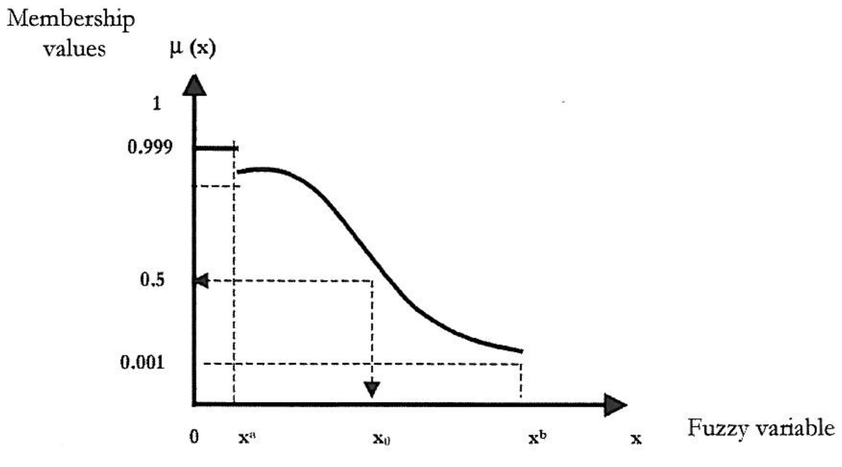

Fig 2: Modified S-Curve membership function

By substituting equation (3) into equation (4):

$$\frac{0.999(1+C)}{1+Ce^{\alpha}} = 0.001 \tag{5}$$

Rearranging equation (5) as,

$$\alpha \;=\; \ln\frac{1}{0.001}\left(\frac{0.998}{C}+0.999\right) \tag{6}$$

Since, B and α depend on C , we require one more condition to get the values for B, C and α.

Let, when $x_0 = \frac{x^a + x^b}{2}$, $\mu(x_0) = 0.5$; Therefore,

$$\frac{B}{1+Ce^{\frac{\alpha}{2}}} = 0.5 \tag{7}$$

and hence,

$$\alpha = 2\ln\left(\frac{2B-1}{C}\right) \tag{8}$$

Substituting equation (3) and equation (6) into equation (8), we obtain

$$2\ln\left(\frac{2(0.999)(1+C)-1}{C}\right) = \ln\frac{1}{0.001}\left(\frac{0.998}{C}+0.999\right) \tag{9}$$

Rearranging equation (9) yields

$$(0.998+1.998C)^2 = C(998+999C) \tag{10}$$

Solving equation (10) :

$$C = \frac{-994.011992 \pm \sqrt{988059.8402+3964.127776}}{1990.015992} \tag{11}$$

Since C has to be positive, equation (11) gives $C = 0.001001001$ and from equation (3) and (6), B = 1 and α = 13.81350956.

Formulation of Fuzzy Product mix Selection Problem (FPSP) with Fuzzy technical Coefficient and Fuzzy Resource variable :

$$\text{Maximize} \quad \sum_{j=1}^{8} c_j x_j \tag{12}$$

$$\text{subject to} \quad \sum_{i=1}^{i=29} \tilde{a}_{ij} x_j \lesssim \tilde{b}_i$$

In equation (1) in which the technical coefficients and the resource variables a_{ij} and b_i respectively are fuzzy quantities, c_j is non-fuzzy objective coefficients and x_j's are decision variables, x > 0. Tabucannon (1996) solved this problem by using a non fuzzy linear programming approach .

3.2 Membership Function The For Resource Variable $\tilde{b}_i$

The membership function for the resource variable is given by :

$$\mu_{b_i} = \begin{cases} 1.000 & b_i < b_i^a \\ 0.999 & b_i = b_i^a \\ \dfrac{B}{1 + Ce^{\alpha\left(\frac{b_i - b_i^a}{b_i^b - c_i^a}\right)}} & b_i^a < b_i < b_i^b \\ 0.001 & b_i = b_i^b \\ 0.000 & b_i > b_i^b \end{cases} \tag{13}$$

where μ_{b_i} is the degree of membership function for the resource variable b_i. b_i^a *and* b_i^b are respectively the lower and the upper boundary for resource variable b_i.

3.2.1 Resource Variable $\tilde{b}_i$

From equation (13) for an interval $b_i^a < b_i < b_i^b$,

$$\mu_{b_i} = \frac{B}{1 + Ce^{\alpha\left(\frac{b_i - b_i^a}{b_i^b - b_i^a}\right)}} \qquad (14)$$

Rearranging exponential term

$$e^{\alpha\left(\frac{b_i - b_i^a}{b_i^b - b_i^a}\right)} = \frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right) \qquad (15)$$

Taking $\log_e$ both sides

$$\alpha\left(\frac{b_i - b_i^a}{b_i^b - b_i^a}\right) = \ln\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right) \qquad (16)$$

Hence

$$b_i = b_i^a + \left(\frac{b_i^b - b_i^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right) \qquad (17)$$

Since b_i is the fuzzy resource variable in equation (17), it is denoted by $\tilde{b}_i$.

Therefore,

$$\tilde{b}_i\Big|_{\mu=\mu_{b_i}} = b_i^a + \left(\frac{b_i^b - b_i^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right) \qquad (18)$$

The membership function for μ_{b_i} and the fuzzy interval, b_i^0 *to* b_i^1 for $\tilde{b}_i$ is given in Figure 3.

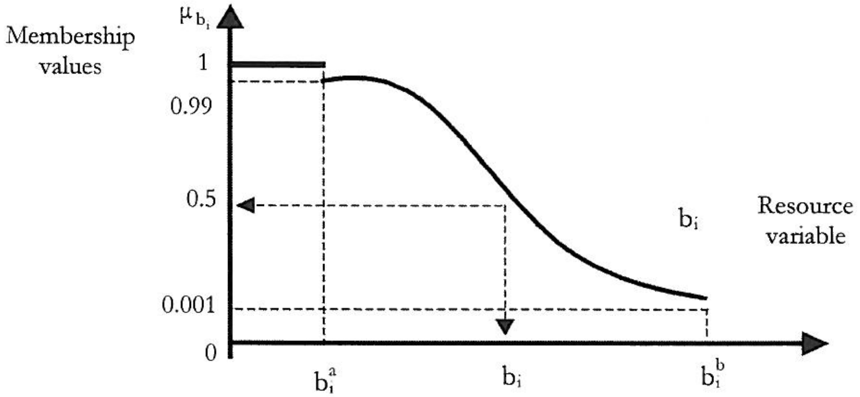

Fig 3: Membership function μ_{b_i} and fuzzy interval for $\tilde{b}_i$

According to Watada (1997), a triangular or trapezoidal membership function shows a necessity level and a sufficiency level at their grades 1 and 0 respectively. On the other hand, considering a non-linear membership function as a flexible S-curve, a necessity level or a sufficiency level may be approximated at the points with grade $\mu_{b_i} = 0.999$ *when* $b_i = b_i^a$ *and* $\mu_{b_i} = 0.001$ *when* $b_i = b_i^b$.

The membership function for technical coefficients can be constructed in a similar form as equation (18).

Using equations (12), (13) and (18), the formulation (12) is made equivalent to :

$$Max \ \sum_{j=1}^{8} c_j x_j$$

Subject to

$$\sum_{i=1}^{29}\left(a_{ij}^a + \left[\frac{a_{ij}^b - a_{ij}^a}{\alpha}\right]\ln\frac{1}{C}\left[\frac{B}{\mu_{a_{ij}}} - 1\right]\right)x_j \le b_i^a + \left[\frac{b_i^b - b_i^a}{\alpha}\right]\ln\frac{1}{C}\left[\frac{B}{\mu_{b_i}} - 1\right]$$

$$where \ \ x_j \ge 0, \quad j = 1,2,3,\ldots,8\,, \quad 0 <, \mu_{a_{ij}}, \mu_{b_i} < 1\,, \quad 0 < \alpha < \infty. \qquad (19)$$

$$\alpha = 13.81350956, \quad C = 0.001001001, \quad B = 1 \quad and \quad 0 < \mu < 1.$$

In equation (19), the best value for the objective function at the fixed level of μ is reached when [2]

$$\mu = \mu_{a_{ij}} = \mu_{b_i} \; for \quad i = 1,2,...,29 \quad , \quad j = 1,2,...,8 \qquad (20)$$

Using equation (18) with the above values of α, B and C, values of b_i are generated and computed for the range $\mu_{c_j} = 0.001$ to $\mu_{c_j} = 0.999$. The interval between two adjacent μ_{c_j} values can be arbitrary but has to be as small as possible to reach a level of precision in optimal solution. Here an interval for μ_{b_i} is considered as 0.0499.

4.0 FLP WITH FUZZY RESOURCES, FUZZY TECHNICAL COEFFICIENTS AND NON FUZZY OBJECTIVE COEFFICIENTS

The Fuzzy Product – mix Selection Problem (FPSP) is stated as:
There are *n* products to be manufactured by mixing *m* raw materials with different proportion and by using *k* varieties of processing. There are limitations in resources of raw materials. There are also some constraints imposed by the marketing department, such as product – mix requirement, main product line requirement and lower and upper limit of demand for each product. All the above requirements and conditions are fuzzy. It is necessary to obtain maximum profit with a certain degree of satisfaction by using interactive fuzzy linear programming.

Chocoman Inc. manufactures 8 chocolate products. There are 8 raw materials to be mixed in different proportions and 9 processes (facilities) to be utilized. The product demand, material and facility available are as illustrated in Table 1 and Table 2 respectively. Table 3 and Table 4 give the mixing proportions and facility usage required for manufacturing each product.

In each Table, the entries are given as fuzzy data with two limits; the lower limit is a crisp data, whereas the upper limit is a fuzzy data, and hence, the range is fuzzy. For example, in Table 1, MC 250 (Milk Chocolate 250 g) has a certainty of 500,000 units of demand. But the range 625,000 – 500,000 = 125,000 is fuzzy. This fuzziness is due to various reasons such as availability and usage of raw material, availability and usage of process facilities, etc. It is needless to indicate that the nature of fuzziness is inevitable in any large manufacturing center such as in Chocoman Inc.

Table 1 : Demand of product

Synonym		Product	Fuzzy Interval ($x10^3$ units)
x_1	MC 250	Milk chocolate, 250 g	[500,625)
x_2	MC 100	Milk chocolate, 100 g	[800,1000)
x_3	CC 250	Crunchy chocolate, 250 g	[400,500)
x_4	CC 100	Crunchy chocolate, 100 g	[600,750)
x_5	CN 250	Chocolate with nuts, 250g	[300,375)
x_6	CN 100	Chocolate with nuts,100 g	[500,625)
x_7	CANDY	Chocolate candy	[200,250)
x_8	WAFER	Wafer	[400,500)

Table 2 : Raw material and facility availability

Raw Material/Facility (units)	Fuzzy Interval for Availability
Coco (kg)	[75000,125000)
Milk (kg)	[90000,150000)
Nuts (kg)	[45000,75000)
Confectionery sugar (kg)	[150000,250000)
Flour (kg)	[15000,25000)
Aluminum foil (ft^2)	[375000,625000)
Paper (ft^2)	[375000,625000)
Plastic (ft^2)	[375000,625000)
Cooking (ton-hours)	[750,1250)
Mixing (ton-hours)	[150,250)
Forming (ton-hours)	[1125,1875)
Grinding (ton-hours)	[150,250)
Wafer making (ton-hours)	[75,125)
Cutting (hours)	[300,500)
Packaging 1 (hours)	[300,500)
Packaging 2 (hours)	[900,1500)
Labor (hours)	[750,1250)

Table 3 : Mixing proportions (Fuzzy)

	Product Types – Fuzzy Interval							
Materials required (per 1000 units)	**MC 250**	**MC 100**	**CC 250**	**CC 100**	**CN 250**	**CN 100**	**CANDY**	**WAFER**
Cocoa (kg)	[66,109)	[26,44)	[56,94)	[22,37)	[37,62)	[15,25)	[45,75)	[9,21)
Milk (kg)	[47,78)	[19,31)	[37,62)	[15,25)	[37,62)	[15,25)	[22,37)	[9,21)
Nuts (kg)	0	0	[28,47)	[11,19)	[56,94)	[22,37)	0	0
Cons. sugar (kg)	[75,125)	[30,50)	[66,109)	[26,44)	[56,94)	[22,37)	[157,262)	[18,30)
Flour (kg)	0	0	0	0	0	0	0	[54,90)
Alum foil (ft^2)	[375,625)	0	[375,625)	0	0	0	0	[187,312)
Paper(ft^2)	[337,562)	0	[337,563)	0	[337,562)	0	0	0
Plastic (ft^2)	[45,75)	[95,150)	[45,75)	[90,150)	[45,75)	[90,150)	[1200,2000)	[187,312)

Table 4 : Facility usage (Fuzzy)

	Product Types – Fuzzy Interval							
Facility usage required (per 1000 units)	**MC 250**	**MC 100**	**CC 250**	**CC 100**	**CN 250**	**CN 100**	**CANDY**	**WAFER**
Cooking (ton-hours)	[0.4,0.6)	[0.1,0.2)	[0.3,0.5)	[0.1,0.2)	[0.3,0.4)	[0.1,0.2)	[0.4,0.7)	[0.1,0.12)
Mixing (ton-hours)	0	0	[0.1,0.2)	[0.04,0.07)	[0.2,0.3)	[0.07,0.12)	0	0
Forming (ton-hours)	[0.6,0.9)	[0.2,0.4)	[0.6,0.9)	[0.2,0.4)	[0.6,0.9)	[0.2,0.4)	[0.7,1.1)	[0.3,0.4)
Grinding (ton-hours)	0	0	[0.2,0.3)	[0.07,0.12)	0	0	0	0
Wafer making (ton-hours)	0	0	0	0	0	0	0	[0.2,0.4)
Cutting (hours)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.15,0.25)	0
Packaging 1(hours)	[0.2,0.3)	0	[0.2,0.3)	0	[0.2,0.3)	0	0	0
Packaging 2(hours)	[0.04,0.06)	[0.2,0.4)	[0.04,0.06)	[0.2,0.4)	[0.04,0.06)	[0.2,0.4)	[1.9,3.1)	[0.1,0.2)
Labor(hours)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[1.9,3.1)	[1.9,3.1)

The following constraints were established by the sales department of Chocoman Inc.:

1. Product mix requirements. Large –sized products (250g) of each type should not exceed 60% (non fuzzy value) of the small-sized product (100 g), such that :

$$x_1 \leq [45\%,75\%)\ x_2 \qquad (21)$$

$$x_3 \leq [45\%,75\%)\ x_4 \qquad (22)$$

$$x_5 \leq [45\%,75\%)\ x_6 \qquad (23)$$

2. Main product line requirement. The total sales from candy and wafer products should not exceed 15% of the total revenues from the chocolate bar products, such that :

$$[300,500)\ x_7 + [112.5, 187.5)\ x_8 \leq [42.19,70.31) x_1 + [16.87,28.12)\ x_2 + [45,77)\ x_3 + [18,30)\ x_4 + [47.25,78.75)\ x_5 + [19.69,32.81)\ x_6 \qquad (24)$$

Table 5 : Objective coefficients

Product	Non Fuzzy Profit ($/100 unit)
Milk chocolate, 250 g	180
Milk chocolate, 100g	83
Crunchy chocolate, 250g	153
Crunchy chocolate, 100g	72
Chocolate with nuts, 250g	130
Chocolate with nuts, 100g	70
Chocolate candy	208
Chocolate wafer	83

By using a linear programming technique, we will be able to solve the above FPSP problem and the best outcome for the objective function can be obtained. The obtained results are summarized in the following tables and figures.

5.0 COMPUTATIONAL PROCEDURE AND SOLUTIONS

The equation (19) is solved by using FLP in MATLAB® and its tool box of LP. This modern technique is far more better than OR Courseware (Hillier, 1995), Elementary Linear Programming with Applications (Kolman and Beck, 1995), and Linear Programming, Introduction software (Thapa, 1997).The vagueness is given by α and the μ is the degree of satisfaction. This tool box has two inputs α and μ in addition to the fuzzy intervals for technical coefficients and resource variables. There is a single output z^*, the optimum profit.

The given values of varies parameters from chocolate manufactures, Chocoman Inc. are fed into the tool box. The solution can be tabulated and presented as a 2D and 3D graph.

The procedure for obtaining the optimal solution for the FLP problem is described as follows :

Step 1 : Set the interval for fuzzy parameter μ from 0 to 1 with interval steps of 0.0499.

Step 2 : For each μ, generate the fuzzy parameter of a_{ij} and b_i , $i = 1, 2, 3, \ldots, 29$ and $j = 1, 2, 3, \ldots, 8$ by using MATLAB® programming

Step 3 : Input the value of fuzzy parameters in Simplex Method of MATLAB® (Optimization Tool Box : Linear Programming) to obtain optimal solution, z^*.

The optimal solution z^* versus membership value μ is plotted by using MATLAB® (2 Dimensional Graphics) as given in Figure 2.

Figure 2 will be presented to the decision maker, then to the implementer for further analysis.

The optimal solution for FPSP is obtained, and it is shown in Table 1 and Figure 2.

The profit function (objective value) has a value 262,000 at $\mu = 1$. We define this as 100% degree of satisfaction. Accordingly, a z^* of value 252,770 has 0.1% degree of satisfaction. The possible realistic solution exists at $\mu = 0.5$ (ie 50% degree of satisfaction) with a value of z^* as 258,360. It is found that z^* has become more than that of a totally non fuzzy situation.

Figure 4 illustrates the variation of objective values z^* with respect to degree of satisfaction μ for one value of vagueness factor $\alpha = 13.81350956$. It will be useful for the decision maker to observe such variations for several values of α.

Objective Values for Various α

Figure 3, shows the nature of variations of z^* with respect to μ when α varies from 2 to 20. The membership value μ in Figure 3 represents the degree of satisfaction and z^* is profit function. We can conclude that when the vagueness increases, the profit value at a particular μ decreases. This phenomenon actually happens in real life problems in a fuzzy system of mix products selection problem.

Table 6: Optimal solutions with respect to degree of satisfaction ($\alpha = 13.81350956$)

Degree of Satisfaction (μ)	Optimal Values z^*
0.0010	254770
0.0509	256840
0.1008	257220
0.1507	257460
0.2006	257640
0.2505	257790
0.3004	257920
0.3503	258040
0.4002	258150
0.4501	258250
0.5000	258360
0.5499	258460
0.5998	258570
0.6497	258680
0.6996	258800
0.7495	258930
0.7994	259070
0.8493	259250
0.8992	259490
0.9491	259870
0.9990	261930

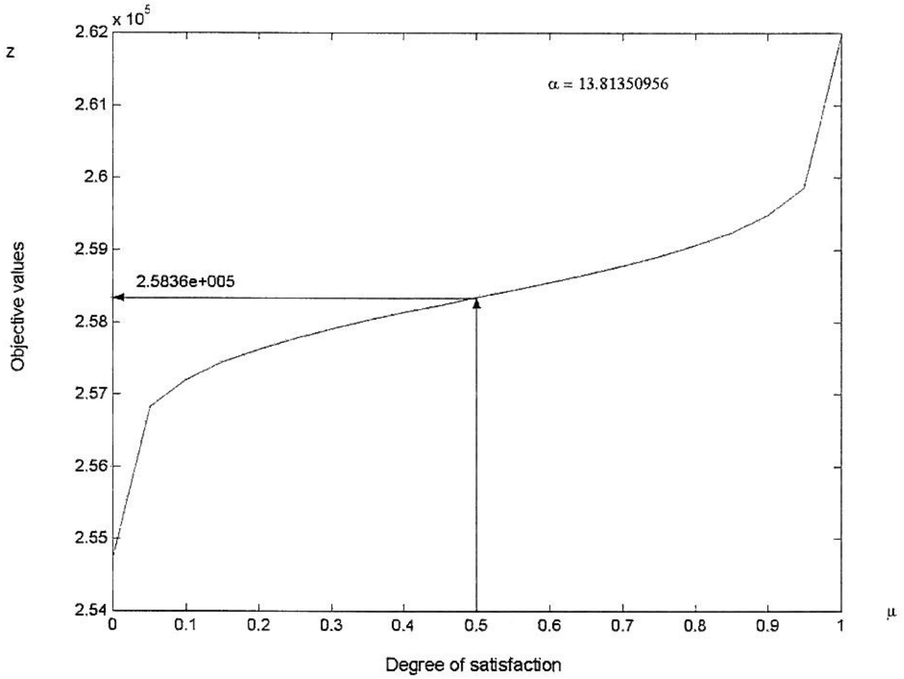

Fig. 4 : Objective values and degree of satisfaction for α=13.81350956

The ideal solution in a fuzzy environment exists at $\mu = 0.5$ (Carlsson and Korhonen, 1986). Hence, the result for 50% degree of satisfaction ($\mu = 0.5$) for $2 \leq \alpha \leq 20$ and the corresponding values for z^* are presented in Table 7.

It can be seen from Table 7 that for $\mu = 0.5$ and as α increases, z^* decreases. It can be concluded that when the vagueness in the variables of objective coefficients increases, z^* decreases with the same degree of satisfaction. The data in Table 7 is the result of analyzing the FLP for equation (19). These data are very useful for the decision maker, who after consulting the implementer, implements a specific decision.

It is found that the logistic membership function with varied values of α and the S-function (with $\alpha = 13.813509$) offer an acceptable solution with a certain degree of satisfaction in the fuzzy system of FPSP. More vagueness results in less profit.

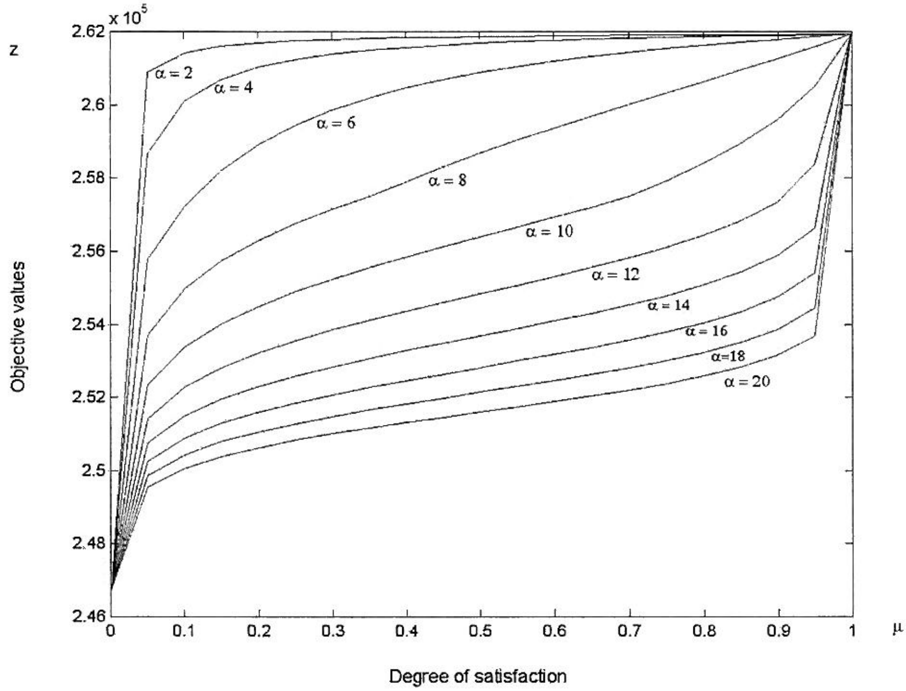

Fig 5 : Degree of satisfaction and objective values for $2 \leq \alpha \leq 20$

In Table 8, we can see that at $z^* = 259{,}000$ when vagueness $\alpha = 2$ then the degree of satisfaction is 2.0%. This shows that if the availability of material, labor and processing happen in a less fuzzy environment (with $\alpha = 2$) then we will obtain a profit of $259,000 at 2.0% degree of satisfaction. Similarly, when vagueness $\alpha = 18$ with a highly fuzzy environment, then we obtain the same profit of $259,000 at 95% degree of satisfaction. Finally, we can conclude that when vagueness increases, the degree of satisfaction also increases at any fixed objective value z^*.

Table 7 : Fuzzy parameter α, and objective values z* (50% degree of satisfaction)

Vagueness α	Objective Value z^*
2	261910
4	261840
6	261530
8	260690
10	259680
12	258880
14	258290
16	257850
18	257510
20	257240

Table 8 : Vagueness and degree of satisfaction at $z^* = 259000$

Vagueness (α)	Degree of Satisfaction (μ)
2	2.00%
4	3.00%
6	4.50%
8	11.0%
10	26.5%
12	53.5%
14	79.0%
16	92.0%
18	95.0%
20	96.0%

The relationship between z^*, μ and α is given in Table 9. This table is very useful for the decision maker to find the profit value at any given value of α with a degree of satisfaction μ. From the table we can see that the objective value is not linearly dependent on the vagueness and the degree of satisfaction. One cannot conclude that for a higher degree of satisfaction, the profit value will be higher. This is not true. But at 100% degree of satisfaction the profit value will be largest, even with the higher value of vagueness. From the diagonal values in the table, we can conclude that the objective value increases at a lower value of μ ($0.001 \leq \mu \leq 0.2505$). Then z^* value decreases for $0.5 \leq \mu \leq 0.7495$. Lastly, z^* value increases for $0.7495 \leq \mu \leq 1$. This result shows that a good decision (higher degree of satisfaction) does not provide a higher value in profit (objective value). This means one should be satisfied with a certain degree of satisfaction when it comes to making decision in a fuzzy environment. The 3D plot for μ , α and z^* is given in Figure 6.

If the obtained membership value of the solution is appropriate and proper, that is, it is included in (0,1), regardless of the shape of a membership function, and whether we employ a linear membership function or a non-linear membership function to the analysis, both solutions do not differ so much. Nevertheless, it is possible that the non-linear membership function changes its shape according to the parameter values. Then a decision maker and implementer are able to apply their strategy to a fuzzy mix product selection problem using these parameters. In this context, the non-linear membership function such as the modified s-curve function is much more convenient than the linear ones.

6.0 CONCLUSION

This paper has illustrated the application of a proposed S-curve membership function in a real world industrial engineering problem, i.e., in the chocolate manufacturing. The real life problem has 8 decision variables with 29 constraints to be included in the FLP formulation. This real life problem has been solved and their results are tabulated. The decision maker can also suggest to the analyst some possible and practicable changes in fuzzy intervals for improving the maximum profit. The analysis and modeling of the fuzzy mix product selection problem presented in this paper is based on precise objective function and fuzzy constraints in the form of non-linear membership functions. The results clearly indicate the superiority of the fuzzy approach in terms of best good enough solutions for the objective functions and degree of satisfaction with respect to vagueness. The work also implies that the modified s-curve membership function once introduced in a fuzzy linear programming model with constraints, provide a similar (or even better) level of degree of satisfaction for obtained results compared to non-fuzzy linear programming. It is possible to design a self organizing fuzzy system for the FPSP in which the analyst, decision maker and the implementers incorporate their knowledge and experience. Future research will be in this direction.

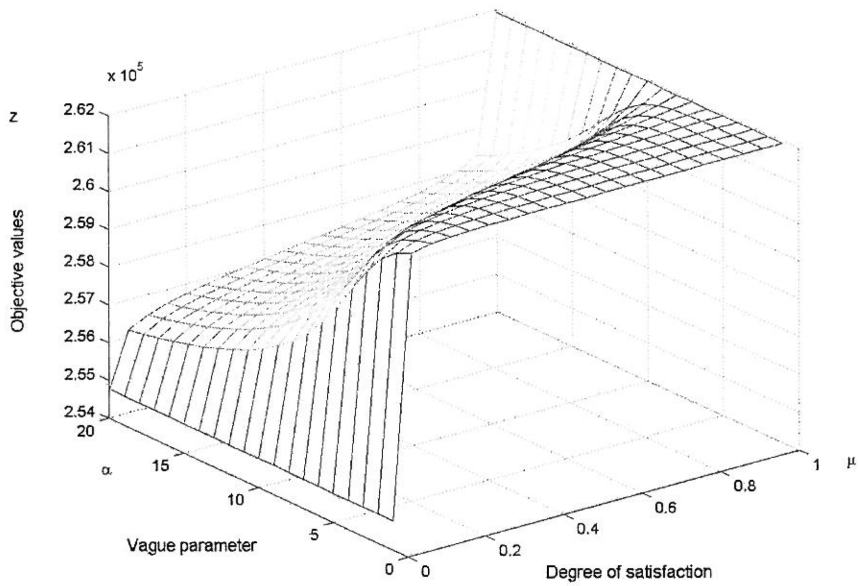

Fig 6 : Degree of Satisfaction μ ,Vagueness α and Objective Values z*

Table 9 : Distribution of z* against μ and α

z*	Vagueness α				
D.S μ	1	5	9	13	17
0.0010	254770	254770	254770	254770	254770
0.2505	261890	261400	259350	257960	257210
0.5000	261920	261730	260170	258570	257670
0.7495	261930	261860	260890	259170	258130
0.9990	206193	261930	261930	261930	261930

Note: D.S = Degree of Satisfaction

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