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Interval Type-2 Gaussian Fuzzy Games: Energy Sharing in Distributed Systems with Electric Vehicle Integration under Uncertainty

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ABSTRACT

Fuzzy cooperative games using Type-1 fuzzy numbers are commonly applied to model vagueness and imprecision in coalition values and player allocations. However, such models are less effective in capturing increased imprecision. This paper introduces interval Type-2 Gaussian fuzzy numbers (IT2 GFNs) to better represent increased imprecision in coalition values and player allocations in distributed energy systems (DES) caused by uncoordinated electric vehicle (EV) charging and discharging patterns. An analytical solution is proposed to ensure fair and efficient player allocations in IT2 Gaussian fuzzy cooperative games. The proposed method avoids the computational challenges of IT2 subtractions and the associated Type-1 fuzzy number subtractions, which tend to amplify the footprint of uncertainty (FOU) or restrict applicability. A comparative analysis with the interval-based Shapley value, computed using Moore's subtraction operator, demonstrates that the proposed method achieves allocation efficiency by fully distributing the grand coalition value among players while overcoming the known limitations of the Hukuhara difference, which is not always well-defined.

Keywords: Distributed energy systems, electric vehicles, interval type-2 Gaussian fuzzy cooperative games, quadratic programming.

INTRODUCTION

Cooperative game theory provides a structured framework for analysing how players collaborate to achieve collective benefits. A cooperative game is defined by a set of players, the grand coalition, and a characteristic function that assigns a crisp value to each subset of players, or coalition (Galindo et al., 2021). Traditional cooperative games assume that coalition values and player allocations are precisely defined, an assumption that often does not hold in real-world applications. For instance, in distributed energy systems (DES), it is difficult to define coalition values and player allocations precisely due to time-of-use energy prices, fluctuating demand, intermittent renewable generation, and storage limitations (Choucair et al., 2025a). This challenge is further compounded by uncoordinated electric vehicle (EV) charging and discharging patterns and differences in battery specifications.

Researchers have developed various methods to address uncertainty in cooperative games. One of the earliest approaches modelled coalition values as random variables with known probability distributions (Charnes & Granot, 1976; Fernández et al., 2002; Suijs et al., 1999). While effective for capturing randomness-based uncertainty, these methods rely on theoretical distributions that may not be available for small-scale distributed energy-sharing systems. Cooperative interval games (Han et al., 2012) were later introduced to estimate coalition values within predefined ranges; however, these require players to know the exact interval boundaries of their coalitions. In the context of EV integration, uncertainty in coalition values has also been addressed using stochastic methods. Aghajani and Kalantar (2017) employed Monte Carlo simulations (Hamundu et al., 2012) to generate uncertainty scenarios for EV charging and discharging, while Sheikhi et al. (2013) developed a stochastic model in which EV arrival times and charging durations were treated as random variables with given probability distributions.

Alternatively, fuzzy cooperative games, in which coalition values are represented by fuzzy numbers (Sharif et al., 2019), offer an effective means of modelling vagueness or imprecision (Mareš, 2000, 2001). Although Type-1 Gaussian fuzzy numbers (T1-GFNs) have been used to model imprecision in coalition values for distributed energy-sharing scenarios (Choucair et al., 2025a), they are less effective in capturing the heightened imprecision caused by EV integration in DES. Castillo et al. (2007) stated that Type-2 fuzzy sets (T2-FS) provide a superior framework for modelling uncertainty and imprecision, particularly when the exact membership function (MF) of a Type-1 fuzzy set (T1-FS) cannot be determined (Mendel, 2007). In T2-FS, each element has a membership value that is itself a fuzzy set within the interval [0, 1], whereas in T1-FS, the membership value is a single, precise number within that range (Dereli et al., 2011). Recently, Type-2 fuzzy models have gained increasing attention and have been applied across various domains (De et al., 2022). Maiti et al. (2025) applied a Gaussian Type-2 fuzzy cooperative game to address the multi-drug resistance problem, in which uncertainty arises from insufficient data that vary over time, by source, or both.

While T2-FS are more adept at capturing uncertainty than T1-FS (De et al., 2022), their three-dimensional (3D) secondary MFs often complicate analysis and operations. A practical alternative is the interval Type-2 fuzzy set (IT2-FS) (Tolga, 2020), which offers a simple and efficient approach to data analysis under uncertainty. IT2-FS have been applied across various domains and provides the advantage of being general enough to model available approximate knowledge (Agüero & Vargas, 2007). In IT2-FS, the secondary membership grade is always one (Mendel et al., 2006), meaning the third dimension conveys no additional information and can be omitted. An IT2-FS is fully characterised by its footprint of uncertainty (FOU), defined by two Type-1 MFs: an upper and a lower function. This structure makes IT2-FS a simplified subclass that retains the ability to model uncertainty while enabling more efficient computation, often based on Type-1 mathematics (Mendel et al., 2006). Mencattini et al.

(2006) utilised IT2-FS to model measurement uncertainty when probabilistic assumptions, such as known probability density functions, are unreliable or unavailable. Agüero and Vargas (2007) employed IT2 fuzzy numbers to enhance fault current calculations in distribution systems by modelling approximate knowledge derived from uncertain data.

Several researchers have implemented IT2-FS in fuzzy logic controllers to address uncertainties associated with EVs. Aljohani (2024) developed an interval Type-2 fuzzy controller to navigate the challenges posed by uncoordinated EV charging and fluctuating renewable energy on the grid. This approach proved particularly effective in dynamic and stochastic environments where incomplete information or system anomalies hinder accurate mapping of MFs. Similarly, Phan et al. (2021) demonstrated that interval Type-2 fuzzy logic controllers outperformed their Type-1 counterparts under uncertain and ambiguous road conditions, managing energy consumption and extending battery life in hybrid and autonomous EVs. Zhang (2021) further contributed by applying an interval Type-2 fuzzy logic controller to address the spatial and temporal uncertainties associated with uncoordinated EV charging.

While IT2-FS with piecewise linear MFs are easier to analyse and work with, Gaussian MFs offer several advantages (Wu & Mendel, 2019). They encompass the entire input domain and require fewer parameters. An IT2 Gaussian fuzzy set is typically formed by blurring either the mean or the standard deviation of a baseline Type-1 Gaussian fuzzy set, requiring only three to four parameters for definition. In contrast, constructing a piecewise linear IT2-FS usually involves more parameters, making it more complex. Moreover, IT2-FS are frequently employed, as they ensure the continuity of the resulting system and facilitate the computation of derivatives with respect to their parameters in gradient-based optimisation algorithms. Additionally, Mendel (2005) demonstrated that for symmetrical IT2-FS, the type adopted in this research, the centroid equals the mean, thereby simplifying the defuzzification process and enhancing computational efficiency.

In this paper, we extend the work of Choucair et al. (2025a) by integrating EVs into a residential community equipped with a DES. To address the increased uncertainty in DES arising from uncoordinated EV charging and discharging patterns and varying battery specifications, we propose using IT2-FS with Gaussian MFs, also known as interval Type-2 Gaussian fuzzy numbers (IT2-GFNs) (Tolga et al., 2020). This approach is suitable for modelling imprecision in coalition values and player allocations when only approximate knowledge or insufficient data is available, making it challenging to define exact MFs. We derive an analytical solution for fuzzy cooperative games with coalition values based on IT2-GFNs. Our method employs quadratic programming to achieve a fair allocation by minimising discrepancies between coalition values and the corresponding player allocations (Zhao & Liu, 2018), and uses Lagrange multipliers to ensure an efficient allocation in which the grand coalition value equals the sum of individual allocations. A key advantage of the proposed approach is that it avoids the computationally expensive subtraction of IT2-GFNs and circumvents the problematic subtraction associated with Type-1 fuzzy numbers, which are known to distort uncertainty and reduce decision accuracy (Choucair et al., 2025a; Zhao & Liu, 2018).

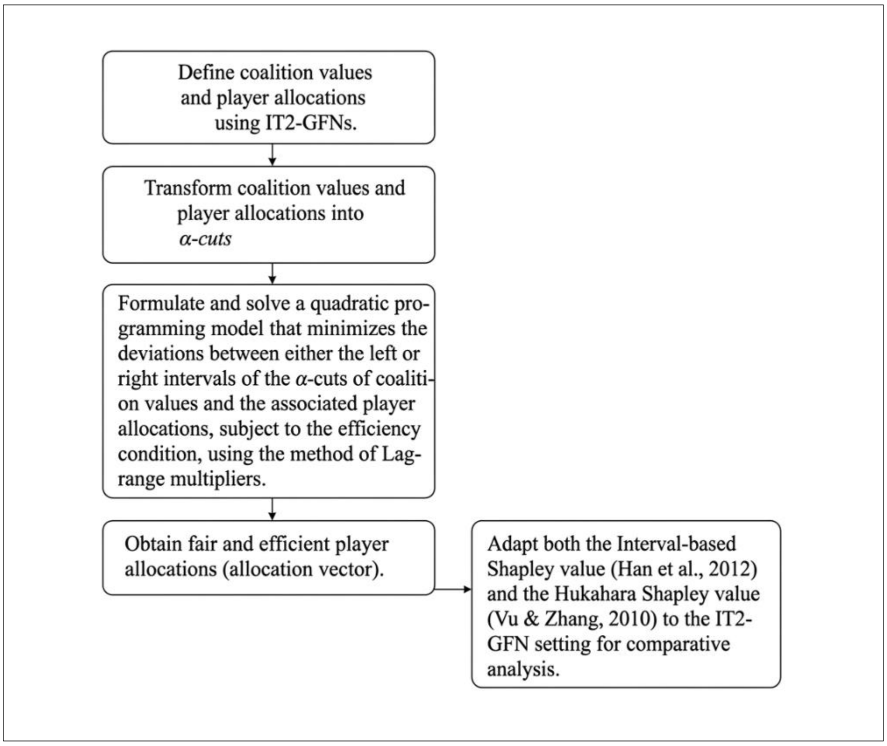
This study offers two main contributions. The first introduces an interval Type-2 Gaussian fuzzy cooperative game to model the increased imprecision arising from integrating EVs into DES, where uncoordinated charging and discharging patterns heighten uncertainty. The second eliminates the need for subtraction operations between IT2-GFNs, which, according to Mendel et al. (2006), can be reduced to Type-1 mathematics and are applied at the upper (UMF) and lower (LMF) MF bounds. Conventional Moore's subtraction amplifies the FOU width, while the Hukuhara difference exists only under restrictive nesting conditions. Therefore, by avoiding these subtraction operations, the proposed approach preserves efficiency and yields realistic variance estimates compared to α-cut methods based on Moore's subtraction (Choucair et al., 2025b). Moreover, it remains applicable even when the Hukuhara difference is undefined.

METHODOLOGY

We begin by defining IT2-GFNs and their α-cuts, noting that an IT2-GFN can be determined from either the left or right interval of its α-cut. Coalition values and player allocations are then expressed in terms of α-cut intervals. To minimise discrepancies between coalition values and their corresponding player allocations—while satisfying the efficiency condition (i.e., the sum of individual allocations equals the grand coalition value)—we employ a quadratic programming model in conjunction with the method of Lagrange multipliers. The optimisation process minimises deviations between the bounds on either the left or right sides of the α-cut intervals. Data on demand, renewable energy generation, and energy prices are sourced from Choucair et al. (2025a), while data on EVs are obtained from Ziad et al. (2019). The characteristic function proposed in Choucair et al. (2025a) is also used to compute coalition values in the presence of EVs. Finally, we adapt both the interval-based Shapley value (Han et al., 2012) and the Hukuhara Shapley value (Yu & Zhang, 2010) to the IT2-GFN framework for comparative analysis. Figure 1 illustrates the flow algorithm for IT2-GFN-based allocation.

Figure 1

Flowchart Illustrating the IT2-GFN-Based Allocation Method



Preliminaries

Definition 1. Type-2 Fuzzy Sets (T2-FS). A T2-FS $\tilde{X}$ over a universe of discourse X is characterised by a Type-2 MF $\mu_{\tilde{X}}(x, u)$, where x is the primary variable and $u \in [0,1]$ is the secondary variable, defined over the domain $J_x \subseteq [0,1]$ (Mendel, 2006) as stated in Equation 1. Formally,

$$\tilde{X} = \{(x, u), \mu_{\tilde{X}}(x, u) | \ x \in X, u \in J_x \subseteq [0,1] \}, \quad (1)$$

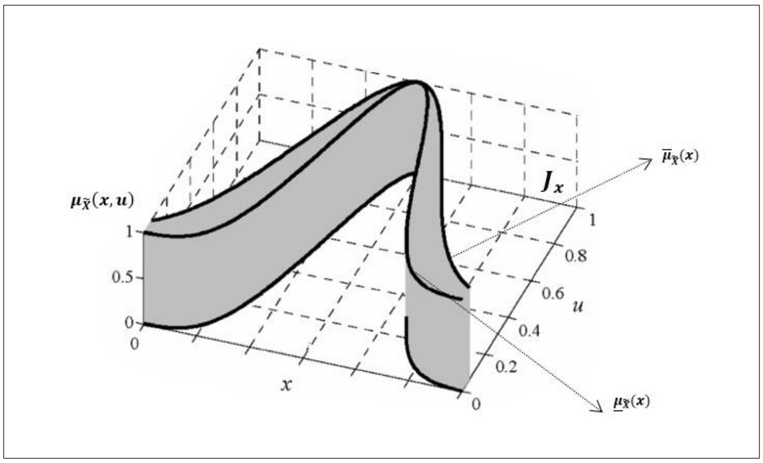
where

$$0 \leq \mu_{\tilde{X}}(x, u) \leq 1.$$

A T2-FS is 3D, as illustrated in Figure 2 (Agüero & Vargas, 2007).

Figure 2

3D Representation of T2-FS



Definition 2. Interval Type-2 Fuzzy Sets (IT2-FS). An IT2-FS is a special case of T2-FS in which all secondary membership grades satisfy $\mu_{\tilde{X}}(x, u) = 1$.

Since $\mu_{\tilde{X}}(x, u) = 1$, the uncertainty in an IT2-FS is entirely captured by the primary membership interval J_x associated with each $x \in X$ (Mendel, 2006).

Definition 3. Footprint of Uncertainty (FOU). The FOU of an IT2-FS $\tilde{X}$ is the region formed by the union of all primary membership intervals J_x associated with each $x \in X$ (Mendel, 2006). It is defined in Equation 2.

$$\mathrm{FOU}(\tilde{X}) = \bigcup_{x \in X} J_x. \quad (2)$$

The FOU, represented by the shaded region in Figure 2, provides a complete description of an IT2-FS. This region is bounded by two Type-1 MFs: the UMF $\overline{\mu}_{\tilde{X}}(x)$ and the LMF $\underline{\mu}_{\tilde{X}}(x)$.
In an IT2-FS, each primary membership interval is defined in Equation 3.

$$J_x = \left[\underline{\mu}_{\tilde{X}}(x), \overline{\mu}_{\tilde{X}}(x)\right], \tag{3}$$

and therefore, the FOU can be expressed in Equation 4.

$$\mathrm{FOU}(\tilde{X}) = \bigcup_{x \in X} \left[\underline{\mu}_{\tilde{X}}(x), \overline{\mu}_{\tilde{X}}(x)\right]. \tag{4}$$

In cases where an IT2-FS employs Type-1 Gaussian MFs with a common mean μ and interval spread $[\underline{\sigma}, \bar{\sigma}]$, the UMF and LMF are defined in Equations 5 and 6 (Castillo, 2012).

$$\overline{\mu}_{\tilde{X}}(x) = exp\left(\frac{-(x-\mu)^2}{2\overline{\sigma}^2}\right) \tag{5}$$

$$\underline{\mu}_{\tilde{X}}(x) = exp\left(\frac{-(x-\mu)^2}{2\underline{\sigma}^2}\right) \tag{6}$$

An IT2-FS of this type is commonly referred to as an IT2-GFN (Tolga et al., 2020).

Definition 4. Perfectly Normal IT2-FS. An IT2-FS $\tilde{X}$ is said to be perfectly normal (Dymova et al., 2015) if it achieves the property in Equation 7.

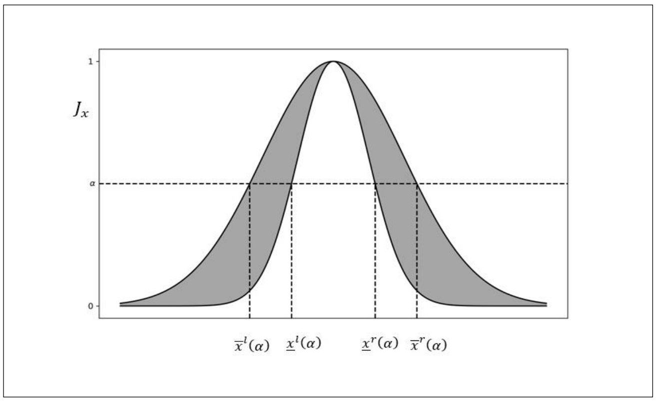
$$\sup_{x \in X} \underline{\mu}_{\tilde{X}}(x) = \sup_{x \in X} \overline{\mu}_{\tilde{X}}(x) = 1 \tag{7}$$

where sup denotes the supremum (the least upper bound) of the membership functions over the universe of discourse X.

Figure 3 illustrates a perfectly normal IT2-FS or a perfectly normal IT2-GFN with a common mean μ and interval spread $[\underline{\sigma}, \bar{\sigma}]$.

Figure 3

2D Representation of Perfectly Normal IT2-FS



Definition 5. α-Cuts of IT2-FS. The α-cut of an IT2-FS $\widetilde{X}$ is defined in Equation 8 (Dymova et al., 2015).

$$\tilde{X}(\alpha) = \left\{x \middle| \underline{\mu}_{\tilde{X}}(x) \geq \alpha, \overline{\mu}_{\tilde{X}}(x) \geq \alpha\right\}, \tag{8}$$

where $\alpha \in (0,1]$.

Since the α-cuts of Type-1 fuzzy numbers are real-valued intervals, the α-cuts of a perfectly normal IT2-FS can be represented as intervals with interval-valued bounds (see Figure 3) as defined in Equations 9 and 10.

$$\tilde{X}(\alpha) = \left[\underline{X}(\alpha), \overline{X}(\alpha)\right] \tag{9}$$

$$= \left[[\overline{x}^l(\alpha), \underline{x}^l(\alpha)], \left[\underline{x}^r(\alpha), \overline{x}^r(\alpha)\right]\right]. \tag{10}$$

Therefore, a perfectly normal IT2-FS can be represented by Equation 11 (Dymova et al., 2015).

$$\tilde{X} = \bigcup\nolimits_{\alpha \in (0,1]} \alpha \left[[\overline{x}^l(\alpha), \underline{x}^l(\alpha)], \left[\underline{x}^r(\alpha), \overline{x}^r(\alpha)\right]\right]. \tag{11}$$

In this paper, we focus exclusively on perfectly normal IT2-FSs, specifically IT2-GFNs with a common mean μ and interval spread $\left[\underline{\sigma}, \bar{\sigma}\right]$.

Based on Definition 5 and the method proposed by Hooda and Raich (2016), we compute the α-cut of an IT2-GFN from Equation 12:

$$exp\left(\frac{-(x-\mu)^2}{2\overline{\sigma}^2}\right) = \alpha\ . \tag{12}$$

Solving Equation 12 for x gives the outcomes in Equation 13.

$$\overline{x}^l(\alpha) = \mu - \bar{\sigma}\sqrt{2\,ln\left(\frac{1}{\alpha}\right)},\ \ \overline{x}^r(\alpha) = \mu + \bar{\sigma}\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}. \tag{13}$$

Similarly, solving Equation 14:

$$exp\left(\frac{-(x-\mu)^2}{2\underline{\sigma}^2}\right) = \alpha\ , \tag{14}$$

we obtain the results in Equation 15.

$$\underline{x}^l(\alpha) = \mu - \underline{\sigma}\sqrt{2\,ln\left(\frac{1}{\alpha}\right)},\ \ \underline{x}^r(\alpha) = \mu + \underline{\sigma}\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}. \tag{15}$$

According to Equations 9, 10, 13, and 15 we observe that an IT2-GFN $\tilde{X}$ can be determined using either bound of the lower interval $\underline{X}(\alpha)$ (left interval) or upper interval $\overline{X}(\alpha)$ (right interval) of the α-cut of $\tilde{X}(\alpha)$. Therefore, in this paper, we focus only on $\underline{x}^r(\alpha)$ of $\overline{X}(\alpha)$ and $\overline{x}^r(\alpha)$ of $\overline{X}(\alpha)$ for determining an IT2-GFN.

A Quadratic Programming Model for Cooperative Games with Coalition Values Based on IT2-GFNs

Consider a fuzzy cooperative game $\tilde{G}(N, \tilde{v})$ defined on the set of players $N = \{1,2,3, \dots, n\}$ with a fuzzy characteristic function $\tilde{v}$. This function assigns an IT2-GFN to each coalition, meaning that $\tilde{v}(S)$ is an IT2-GN for all $S \subseteq N$, and $\tilde{v}(\emptyset) = 0$.

Let $\tilde{X}_i$ represent the allocation of the player $i \in N$. Denote by $\tilde{v}(S)(\alpha)$ the α-cut for all $S \subseteq N$, and let $\tilde{X}_i(\alpha)$ be the α-cut of the allocation of the player $i \in N$. According to Equations 9 and 10, we obtain Equations 16, 17 and 18.

$$\tilde{v}(S)(\alpha) = \left[\underline{v}(S)(\alpha), \overline{v}(S)(\alpha)\right] \tag{16}$$

where

$$\underline{v}(S)(\alpha) = \left[\overline{v}^l(S)(\alpha), \underline{v}^l(S)(\alpha)\right] \tag{17}$$

and

$$\overline{v}(S)(\alpha) = \left[\underline{v}^r(S)(\alpha), \overline{v}^r(S)(\alpha)\right]. \tag{18}$$

Similarly, the allocation of player i leads to Equations 19, 20 and 21.

$$\tilde{X}_i(\alpha) = \left[\underline{X_i}(\alpha), \overline{X}_i(\alpha)\right] \tag{19}$$

where

$$\underline{X}_i(\alpha) = \left[\overline{x}^l_i(\alpha), \underline{x}^l_i(\alpha)\right] \tag{20}$$

and

$$\overline{X}_i(\alpha) = \left[\underline{x_i^r}(\alpha), \overline{x}_i^r(\alpha)\right]. \tag{21}$$

Based on interval arithmetic as defined by Moore (1979), we have Equation 22.

$$\sum_{i \in S} \overline{X}_i(\alpha) = \left[\sum_{i \in S} \underline{x_i^r}(\alpha), \sum_{i \in S} \overline{x}_i^r(\alpha)\right]. \tag{22}$$

To solve such a game, i.e., find $\tilde{X}_i$ for all $i \in N$, we seek to minimise the sum of the square distances between $\sum_{i\in S} \underline{x}_i^r(\alpha)$ and $\underline{v}^r(S)(\alpha)$ and between $\sum_{i\in S} \overline{x}_i^r(\alpha)$ and $\overline{v}^r(S)(\alpha)$ for all $i \in S$ and $S \subseteq N$, $\alpha \in (0,1]$ (Zhao & Liu, 2018). Hence, we aim to minimise the objective function in Equation 23 subject to Equations 24 and 25.

$$\sum_{S\subseteq N} \left[\left(\sum_{i\in S} \underline{x}_i^r(\alpha) - \underline{v}^r(S)(\alpha)\right)^2 + \left(\sum_{i\in S} \overline{x}_i^r(\alpha) - \overline{v}^r(S)(\alpha)\right)^2 \right] \tag{23}$$

subject to the efficiency constraints:

$$\sum_{i\in S} \underline{x}_i^r(\alpha) = \underline{v}^r(N)(\alpha) \tag{24}$$

and

$$\sum_{i\in S} \overline{x}_i^r(\alpha) = \overline{v}^r(N)(\alpha). \tag{25}$$

Let f denote the objective function, and g and h denote the efficiency constraints. This formulation leads to Equations 26, 27 and 28.

$$f = \sum_{S\subseteq N} \left[\left(\sum_{i\in S} \underline{x}_i^r(\alpha) - \underline{v}^r(S)(\alpha)\right)^2 + \left(\sum_{i\in S} \overline{x}_i^r(\alpha) - \overline{v}^r(S)(\alpha)\right)^2 \right] \tag{26}$$

$$g = \sum_{i\in S} \underline{x}_i^r(\alpha) - \underline{v}^r(N)(\alpha) \tag{27}$$

and

$$h = \sum_{i\in S} \overline{x}_i^r(\alpha) - \overline{v}^r(N)(\alpha). \tag{28}$$

To minimize f subject to the efficiency constraints, we use the Lagrange multiplier method (Bertsekas, 1982). Thus, we seek to solve Systems 29 and 30 for all $i \in N$.

$$\begin{cases} \frac{\partial f}{\partial \underline{x}_i^r(\alpha)} = \lambda \frac{\partial g}{\partial \underline{x}_i^r(\alpha)} \\ g = 0 \end{cases} \tag{29}$$

$$\begin{cases} \frac{\partial f}{\partial \overline{x}_i^r(\alpha)} = \tau \frac{\partial h}{\partial \overline{x}_i^r(\alpha)} \\ h = 0 \end{cases} \tag{30}$$

Starting with System (29), the condition $\frac{\partial f}{\partial \underline{x}_i^r(\alpha)} = \lambda \frac{\partial g}{\partial \underline{x}_i^r(\alpha)}$ implies that

$$2 \sum_{S\subseteq N} \left(\sum_{i\in S} \underline{x}_i^r(\alpha) - \underline{v}^r(S)(\alpha)\right) = \lambda. \tag{31}$$

Iterating Equation 31 and utilising Equation 24, we derive the linear System 32 for all $S \neq N$.

$$\begin{cases} n\underline{x}_1^r(\alpha)+\underline{x}_2^r(\alpha)+\underline{x}_3^r(\alpha)+\cdots+\underline{x}_n^r(\alpha)=\frac{\lambda}{2}+\sum_{1\in S}\underline{v}^r(S)(\alpha) \\ \underline{x}_1^r(\alpha)+n\underline{x}_2^r(\alpha)+\underline{x}_3^r(\alpha)+\cdots+\underline{x}_n^r(\alpha)=\frac{\lambda}{2}+\sum_{2\in S}\underline{v}^r(S)(\alpha) \\ \underline{x}_1^r(\alpha)+\underline{x}_2^r(\alpha)+n\underline{x}_3^r(\alpha)+\cdots+\underline{x}_n^r(\alpha)=\frac{\lambda}{2}+\sum_{3\in S}\underline{v}^r(S)(\alpha) \\ \vdots \\ \underline{x}_1^r(\alpha)+\underline{x}_2^r(\alpha)+\underline{x}_3^r(\alpha)+\cdots+n\underline{x}_n^r(\alpha)=\frac{\lambda}{2}+\sum_{n\in S}\underline{v}^r(S)(\alpha) \end{cases} \quad (32)$$

Summing all the equations in System 32 , we obtain:

$$(2n-1)\underline{x}_1^r(\alpha)+(2n-1)\underline{x}_2^r(\alpha)+(2n-1)\underline{x}_3^r(\alpha)+\cdots+(2n-1)\underline{x}_n^r(\alpha) \\ =n\frac{\lambda}{2}+\sum_{1\in S}\underline{v}^r(S)(\alpha)+\sum_{2\in S}\underline{v}^r(S)(\alpha)+\sum_{3\in S}\underline{v}^r(S)(\alpha)+\ldots+\sum_{n\in S}\underline{v}^r(S)(\alpha). \quad (33)$$

Factoring out $(2n-1)$, Equation 33 simplifies to:

$$(2n-1)\left[\underline{x}_1^r(\alpha)+\underline{x}_2^r(\alpha)+\underline{x}_3^r(\alpha)+\cdots+\underline{x}_n^r(\alpha)\right] \\ = n\frac{\lambda}{2}+\sum_{1\in S}\underline{v}^r(S)(\alpha)+\sum_{2\in S}\underline{v}^r(S)(\alpha)+\sum_{3\in S}\underline{v}^r(S)(\alpha)+\ldots+\sum_{n\in S}\underline{v}^r(S)(\alpha). \quad (34)$$

Notice that $\underline{x}_1^r(\alpha)+\underline{x}_2^r(\alpha)+\underline{x}_3^r(\alpha)+\cdots+\underline{x}_n^r(\alpha) = \underline{v}^r(N)(\alpha)$ from Equation 24 ;we can therefore rewrite Equation 34 using Equation 35 as:

$$(2n-1)\,\underline{v}^r(N)(\alpha)=n\frac{\lambda}{2}+\sum_{1\in S}\underline{v}^r(S)(\alpha)+\sum_{2\in S}\underline{v}^r(S)(\alpha)+ \\ \sum_{3\in S}\underline{v}^r(S)(\alpha)+\ldots+\sum_{n\in S}\underline{v}^r(S)(\alpha). \quad (35)$$

Now, solving for λ, we get Equation 36 :

$$\lambda=\frac{2(2n-1)}{n}v^l(N)(\alpha)-\frac{2}{n}\Big[\sum_{1\in S}\underline{v}^r(S)(\alpha)+\sum_{2\in S}\underline{v}^r(S)(\alpha)+ \\ \sum_{3\in S}\underline{v}^r(S)(\alpha)+\ldots+\sum_{n\in S}\underline{v}^r(S)(\alpha)\Big]. \quad (36)$$

Replacing λ in the first equation of System 32 , we obtain Equation 37.

$$n\underline{x}_1^r(\alpha)+\underline{x}_2^r(\alpha)+\underline{x}_3^r(\alpha)+\cdots+\underline{x}_n^r(\alpha)=\frac{(2n-1)}{n}\,\underline{v}^r(N)(\alpha)- \\ \frac{1}{n}\Big[\sum_{1\in S}\underline{v}^r(S)(\alpha)+\sum_{2\in S}\underline{v}^r(S)(\alpha)+\sum_{3\in S}\underline{v}^r(S)(\alpha)+\ldots+\sum_{n\in S}\underline{v}^r(S)(\alpha)\Big]+\sum_{1\in S}\underline{v}^r(S)(\alpha). \quad (37)$$

After further simplification of the right-hand side of Equation 37, we can easily observe Equation 38.

$$n\underline{x}_1^r(\alpha)+\underline{x}_2^r(\alpha)+\underline{x}_3^r(\alpha)+\cdots+\underline{x}_n^r(\alpha)=\frac{(2n-1)}{n}\,\underline{v}^r(N)(\alpha)+\frac{1}{n}\Big[\sum_{1\in S}(|N|- \\ |S|)\underline{v}^r(S)(\alpha)\Big]-\frac{1}{n}\Big[\sum_{1\notin S}|S|\underline{v}^r(S)(\alpha)\Big]. \quad (38)$$

Once λ is substituted into the remaining equations of System 32, the resulting system can be expressed by Equations 39, 40, 41 and 42.

$$A\underline{\boldsymbol{X}}^{\boldsymbol{r}}(\alpha) = \underline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha). \tag{39}$$

Where:

$$A = \begin{pmatrix} n & 1 & \dots & 1 \\ 1 & n & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & n \end{pmatrix}, \tag{40}$$

$$\underline{\boldsymbol{X}}^{\boldsymbol{r}}(\alpha) = \begin{pmatrix} \underline{x}_1^r(\alpha) \\ \underline{x}_2^r(\alpha) \\ \underline{x}_3^r(\alpha) \\ . \\ . \\ . \\ \underline{x}_n^r(\alpha) \end{pmatrix}, \tag{41}$$

and

$$\underline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha) = \begin{pmatrix} \frac{(2n-1)}{n}\ \underline{v}^r(N)(\alpha) + \frac{1}{n}\left[\sum_{1\in S}(n-|S|)\underline{v}^r(S)(\alpha)\right] - \frac{1}{n}\left[\sum_{1\notin S}|S|\underline{v}^r(S)(\alpha)\right] \\ \frac{(2n-1)}{n}\ \underline{v}^r(N)(\alpha) + \frac{1}{n}\left[\sum_{2\in S}(n-|S|)\underline{v}^r(S)(\alpha)\right] - \frac{1}{n}\left[\sum_{2\notin S}|S|\underline{v}^r(S)(\alpha)\right] \\ \frac{(2n-1)}{n}\ \underline{v}^r(N)(\alpha) + \frac{1}{n}\left[\sum_{3\in S}(n-|S|)\underline{v}^r(S)(\alpha)\right] - \frac{1}{n}\left[\sum_{3\notin S}|S|\underline{v}^r(S)(\alpha)\right] \\ . \\ . \\ . \\ \frac{(2n-1)}{n}\ \underline{v}^r(N)(\alpha) + \frac{1}{n}\left[\sum_{n\in S}(n-|S|)\underline{v}^r(S)(\alpha)\right] - \frac{1}{n}\left[\sum_{n\notin S}|S|\underline{v}^r(S)(\alpha)\right] \end{pmatrix}. \tag{42}$$

To demonstrate that Equation 39 has a unique solution $\underline{\boldsymbol{X}}^{\boldsymbol{r}}(\alpha) = A^{-1}\underline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha)$, we need to show that matrix A is invertible (nonsingular) based on the following theorem (Moldovan & Gowda, 2009).

Theorem 1. Strict Diagonal Dominance Theorem. Let $A = (a_{ij})$ be an $n \times n$ complex (or real) matrix. The matrix A is invertible if it is strictly diagonally dominant, i.e., for every row i $(1 \le i \le n)$, Inequality 43 must hold.

$$|a_{ij}| > \sum_{\substack{j=1 \\ j\neq i}} |a_{ij}| \tag{43}$$

The diagonal elements of A are $a_{ii} = n$ for $1 \le i \le n,$ $(n \ge 2)$, while off-diagonal elements are equal to 1. If we sum across each row i, excluding the diagonal element a_{ii}, we obtain:

$$\sum_{\substack{j=1 \\ j\neq i}} |a_{ij}| = 1 + 1 + 1 \dots + 1 = n - 1$$

Clearly, $n > n - 1$, for all $n \ge 2$. Hence, Inequality 43 holds and therefore A is invertible.

Applying A^{-1} to Equation 39 yields the unique vector solution as represented by Equations 44 and 45.

$$\underline{\boldsymbol{X}}^{\boldsymbol{r}}(\boldsymbol{\alpha}) = A^{-1}\underline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha). \tag{44}$$

Where:

$$A^{-1} = \begin{pmatrix} \frac{2}{2n-1} & \frac{-1}{(2n-1)(n-1)} & \cdots & \frac{-1}{(2n-1)(n-1)} \\ \frac{-1}{(2n-1)(n-1)} & \frac{2}{2n-1} & \cdots & \frac{-1}{(2n-1)(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-1}{(2n-1)(n-1)} & \frac{-1}{(2n-1)(n-1)} & \cdots & \frac{2}{2n-1} \end{pmatrix}. \tag{45}$$

To find $\overline{\boldsymbol{X}}^{\boldsymbol{r}}(\alpha)$, proceed to solve System 30:

$$\begin{cases} \frac{\partial f}{\partial \overline{x}_i^r(\alpha)} = \tau \frac{\partial h}{\partial \overline{x}_i^r(\alpha)} \\ h = 0 \end{cases}$$

The calculation for $\overline{\boldsymbol{X}}^{\boldsymbol{r}}(\alpha)$ follows the same procedure as that for $\underline{\boldsymbol{X}}^{\boldsymbol{r}}(\boldsymbol{\alpha})$. The main steps involve analogous substitutions and solving matric equations. For brevity, we can similarly show Equations 46, 47, and 48.

$$A\overline{\boldsymbol{X}}^{\boldsymbol{r}}(\alpha) = \overline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha) \tag{46}$$

where

$$\overline{\boldsymbol{X}}^{\boldsymbol{r}}(\alpha) = \begin{pmatrix} \overline{x}^r_1(\alpha) \\ \overline{x}^r_2(\alpha) \\ \overline{x}^r_3(\alpha) \\ . \\ . \\ . \\ \overline{x}^r_n(\alpha) \end{pmatrix}, \tag{47}$$

and

$$\overline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha) = \begin{pmatrix} \frac{(2n-1)}{n}\,\overline{v}^r(N)(\alpha) + \frac{1}{n}\left[\sum_{1\in S}(n-|S|)\overline{v}^r(S)(\alpha)\right] - \frac{1}{n}\left[\sum_{1\notin S}|S|\overline{v}^r(S)(\alpha)\right] \\ \frac{(2n-1)}{n}\,\overline{v}^r(N)(\alpha) + \frac{1}{n}\left[\sum_{2\in S}(n-|S|)\overline{v}^r(S)(\alpha)\right] - \frac{1}{n}\left[\sum_{2\notin S}|S|\overline{v}^r(S)(\alpha)\right] \\ \frac{(2n-1)}{n}\,\overline{v}^r(N)(\alpha) + \frac{1}{n}\left[\sum_{3\in S}(n-|S|)\overline{v}^r(S)(\alpha)\right] - \frac{1}{n}\left[\sum_{3\notin S}|S|\overline{v}^r(S)(\alpha)\right] \\ . \\ . \\ . \\ \frac{(2n-1)}{n}\,\overline{v}^r(N)(\alpha) + \frac{1}{n}\left[\sum_{n\in S}(n-|S|)\overline{v}^r(S)(\alpha)\right] - \frac{1}{n}\left[\sum_{n\notin S}|S|\overline{v}^r(S)(\alpha)\right] \end{pmatrix}. \tag{48}$$

Since A is invertible, $A\overline{\boldsymbol{X}}^{\boldsymbol{r}}(\alpha) = \overline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha)$ has a unique solution given by Equation 49.

$$\overline{\boldsymbol{X}}^{\boldsymbol{r}}(\boldsymbol{\alpha}) = A^{-1}\overline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha). \tag{49}$$

Cooperative Energy Sharing in a Small Residential Community

This section examines a small residential community equipped with distributed energy resources and EVs. In this community, a set of players $N = \{1,2,3\}$ collaborates to minimise energy costs and maximise revenue under uncertainty. The players are defined as follows: $i = 1$ represents demand, $i = 2$ represents renewable energy (solar energy), and $i = 3$ represents storage. This scenario is modelled as a cooperative game in which coalition values are represented by IT2-GFNs. The total gain of the grand coalition is allocated using the proposed solution method.

The community includes two EVs, which are scheduled to charge between 12:00 AM and 6:00 AM and discharge between 6:00 PM and 11:59 PM to maximise overall gain under a given pricing strategy. The specifications of the EVs (Ziad et al., 2019) used in this study are summarised in Table 1.

Table 1

EV Details

Total battery capacity	Energy needed to charge (60% of 155 kWh)	Charging duration	Average charging power
155 kWh	93 kWh	6 hrs.	15.5 kw

Data on demand profiles, renewable energy generation, and energy prices are obtained from Choucair et al. (2025a). The characteristic function is also adopted from Choucair et al. (2025a) to compute the mean coalition values (μ), while the corresponding spread parameters $[\underline{\sigma}, \bar{\sigma}]$ are estimated based on expert judgment. Table 2 presents the computed coalition values, expressed in United Arab Emirates Dirham (AED) per month.

Table 2

Coalition Values Represented by IT2-GFNs

Coalition S	μ	$[\underline{\sigma}, \bar{\sigma}]$
$\tilde{v}(\{1\})$	0.00	$[0.00, 0.00]$
$\tilde{v}(\{2\})$	329.70	$[70.00, 72.00]$
$\tilde{v}(\{3\})$	166.35	$[45.00, 50.00]$
$\tilde{v}(\{1,2\})$	383.46	$[70.00, 75.00]$
$\tilde{v}(\{1,3\})$	968.55	$[200.00, 225.00]$
$\tilde{v}(\{2,3\})$	496.05	$[90.00, 100.00]$
$\tilde{v}(N)$	2014.20	$[250.00, 300.00]$

Computing Allocations Based on IT2-GFNs

Based on Equations 13 and 15 $\underline{v}^r(S)(\alpha)$ and $\overline{v}^r(S)(\alpha)$ are presented in Table 3.

Table 3

Right Bounds of the α-Cut of Coalition S

α-Cut *of coalition S*	$\underline{v}^r(S)(\alpha)$	$\overline{v}^r(S)(\alpha)$
$\tilde{v}(\{1\})(\alpha)$	0.00	0.00
$\tilde{v}(\{2\})(\alpha)$	$329.70 + 70.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$	$329.70 \ + 72.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$
$\tilde{v}(\{3\})(\alpha)$	$166.35 + 45.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$	$166.35 + 50.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$
$\tilde{v}(\{1,2\})(\alpha)$	$383.46 + 70.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$	$383.46 + 75.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$
$\tilde{v}(\{1,3\})(\alpha)$	$968.55 + 200.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$	$968.55 + 225.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$
$\tilde{v}(\{2,3\})(\alpha)$	$496.05 + 90.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$	$496.05 + 100.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$
$\tilde{v}(N)(\alpha)$	$2014.20 + 250.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$	$2014.20 + 300.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$

As stated in Equation 42 and for $n = \ 3$:

$$\underline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha) =$$

$$\begin{pmatrix} \frac{5}{3}\ \underline{v}^r(N)(\alpha) + \frac{1}{3}\left[2\underline{v}^r(\{1\})(\alpha) + \underline{v}^r(\{1,2\})(\alpha) + \underline{v}^r(\{1,3\})(\alpha)\right] \\ -\frac{1}{3}\left[\underline{v}^r(\{2\})(\alpha) + \underline{v}^r(\{3\})(\alpha) + 2\underline{v}^r(\{2,3\})(\alpha)\right] \\ \frac{5}{3}\ \underline{v}^r(N)(\alpha) + \frac{1}{3}\left[2\underline{v}^r(\{2\})(\alpha) + \underline{v}^r(\{1,2\})(\alpha) + \underline{v}^r(\{2,3\})(\alpha)\right] \\ -\frac{1}{3}\left[\underline{v}^r(\{1\})(\alpha) + \underline{v}^r(\{3\})(\alpha) + 2\underline{v}^r(\{1,3\})(\alpha)\right] \\ \frac{5}{3}\ \underline{v}^r(N)(\alpha) + \frac{1}{3}\left[2\underline{v}^r(\{3\})(\alpha) + \underline{v}^r(\{1,3\})(\alpha) + \underline{v}^r(\{2,3\})(\alpha)\right] \\ -\frac{1}{3}\left[\underline{v}^r(\{1\})(\alpha) + \underline{v}^r(\{2\})(\alpha) + 2\underline{v}^r(\{1,2\})(\alpha)\right] \end{pmatrix}$$

By substituting $\underline{v}^r(S)(\alpha)$ for all $S \subseteq N$ from Table 3, $\underline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha)$ becomes:

$$\underline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha) = \begin{pmatrix} 3311.62 + 408.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 3168.82 + 368.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 3590.56 + 473.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \end{pmatrix}.$$

According to Equation 45, $A^{-1}{}_{n\times n}(a_{ij})$ is defined by:

$$a_{ij} = \begin{cases} \frac{2}{2n-1} & i = j \\ \frac{-1}{(2n-1)(n-1)} & i \neq j \end{cases}$$

and for $n = 3$:

$$A^{-1} = \begin{pmatrix} \frac{2}{5} & -\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2}{5} & -\frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{10} & \frac{2}{5} \end{pmatrix}.$$

From Equation 44, $\underline{\boldsymbol{X}}^{\boldsymbol{r}}(\boldsymbol{\alpha})$ is given by:

$$\underline{\boldsymbol{X}}^{\boldsymbol{r}}(\boldsymbol{\alpha}) = A^{-1}\underline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha).$$

Therefore,

$$\underline{\boldsymbol{X}}^{\boldsymbol{r}}(\boldsymbol{\alpha}) = \begin{pmatrix} \frac{2}{5} & -\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2}{5} & -\frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{10} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 3311.62 + 408.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 3168.82 + 368.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 3590.56 + 473.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \end{pmatrix}$$

$$= \begin{pmatrix} 648.71 + 79.17\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 577.31 + 59.17\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 788.18 + 111.66\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \end{pmatrix}$$

$$= \begin{pmatrix} \underline{x}^r_1(\alpha) \\ \underline{x}^r_2(\alpha) \\ \underline{x}^r_3(\alpha) \end{pmatrix}.$$

On the other hand, $\overline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha)$ is defined by Equation 48 for $n = 3$, as follows:

$$\overline{\boldsymbol{V}}^{\boldsymbol{r}}(\alpha) =$$

$$\begin{pmatrix} \frac{5}{3}\,\overline{v}^r(N)(\alpha) + \frac{1}{3}\left[2\overline{v}^r(\{1\})(\alpha) + \overline{v}^r(\{1,2\})(\alpha) + \overline{v}^r(\{1,3\})(\alpha)\right] \\ -\frac{1}{3}\left[\overline{v}^r(\{2\})(\alpha) + \overline{v}^r(\{3\})(\alpha) + 2\overline{v}^r(\{2,3\})(\alpha)\right] \\ \frac{5}{3}\,\overline{v}^r(N)(\alpha) + \frac{1}{3}\left[2\overline{v}^r(\{2\})(\alpha) + \overline{v}^r(\{1,2\})(\alpha) + \overline{v}^r(\{2,3\})(\alpha)\right] \\ -\frac{1}{3}\left[\overline{v}^r(\{1\})(\alpha) + \overline{v}^r(\{3\})(\alpha) + 2\overline{v}^r(\{1,3\})(\alpha)\right] \\ \frac{5}{3}\,\overline{v}^r(N)(\alpha) + \frac{1}{3}\left[2\overline{v}^r(\{3\})(\alpha) + \overline{v}^r(\{1,3\})(\alpha) + \overline{v}^r(\{2,3\})(\alpha)\right] \\ -\frac{1}{3}\left[\overline{v}^r(\{1\})(\alpha) + \overline{v}^r(\{2\})(\alpha) + 2\overline{v}^r(\{1,2\})(\alpha)\right] \end{pmatrix}.$$

By substituting $\overline{v}^r(S)(\alpha)$ for all $S \subseteq N$ from Table 3, $\overline{\boldsymbol{V}}^r(\alpha)$ becomes:

$$\overline{\boldsymbol{V}}^r(\alpha) = \begin{pmatrix} 3311.62 + 492.67\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 3168.82 + 439.67\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 3590.56 + 567.67\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \end{pmatrix}.$$

According to Equation 49, $\overline{\boldsymbol{X}}^r(\boldsymbol{\alpha})$ is given by:

$$\overline{\boldsymbol{X}}^r(\boldsymbol{\alpha}) = A^{-1}\overline{\boldsymbol{V}}^r(\alpha).$$

Therefore,

$$\overline{\boldsymbol{X}}^r(\boldsymbol{\alpha}) = \begin{pmatrix} \frac{2}{5} & -\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2}{5} & -\frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{10} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 3311.62 + 492.67\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 3168.82 + 439.67\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 3590.56 + 567.67\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \end{pmatrix}$$

$$= \begin{pmatrix} 648.71 + 96.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 577.31 + 69.83\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \\ 788.18 + 133.84\sqrt{2\,ln\left(\frac{1}{\alpha}\right)} \end{pmatrix}$$

$$= \begin{pmatrix} \overline{x}^r_1(\alpha) \\ \overline{x}^r_2(\alpha) \\ \overline{x}^r_3(\alpha) \end{pmatrix}.$$

Thus, the right bounds of the vectors $\underline{\boldsymbol{X}}^r(\boldsymbol{\alpha})$ and $\overline{\boldsymbol{X}}^r(\boldsymbol{\alpha})$, computed previously, are presented in Table 4.

Table 4

Right Bounds of the α-Cut of Player Allocations

Player i α-cut allocation	$\underline{x}^r_i(\alpha)$	$\overline{x}^r_i(\alpha)$
$\tilde{X}_1(\alpha)$	$648.71 + 79.17\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$	$648.71 + 96.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$
$\tilde{X}_2(\alpha)$	$577.31 + 59.17\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$	$577.31 + 69.83\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$
$\tilde{X}_3(\alpha)$	$788.18 + 111.66\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$	$788.18 + 133.84\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}$

According to Equations 13 and 15, the left bounds of the α-cut intervals for the player allocations can be derived from their corresponding right bounds in Table 4. Table 5 presents these left bounds for each player's α-cut allocation.

Table 5

Left Bounds of the α-Cut of Player Allocations

Player i α-cut allocation	$\overline{x}_i^l(\alpha)$	$\underline{x}_i^l(\alpha)$
$\tilde{X}_1(\alpha)$	$648.71 - 96.33\sqrt{2\ ln\left(\frac{1}{\alpha}\right)}$	$648.71 - 79.17\sqrt{2\ ln\left(\frac{1}{\alpha}\right)}$
$\tilde{X}_2(\alpha)$	$577.31 - 69.83\sqrt{2\ ln\left(\frac{1}{\alpha}\right)}$	$577.31 - 59.17\sqrt{2\ ln\left(\frac{1}{\alpha}\right)}$
$\tilde{X}_3(\alpha)$	$788.18 - 133.84\sqrt{2\ ln\left(\frac{1}{\alpha}\right)}$	$788.18 - 111.66\sqrt{2\ ln\left(\frac{1}{\alpha}\right)}$

Based on Equations 9, 10, 13, and 15, the IT2-GFN allocations are constructed using the values in Table 4 and Table 5. These allocations are presented in Table 6, where μ denotes the mean and $[\bar{\sigma}, \underline{\sigma}]$ represents the interval spread of each IT2-GFN allocation.

Table 6

IT2 GFN-Based Allocations

Player i allocation	μ	$[\underline{\sigma}, \bar{\sigma}]$
$\tilde{X}_1$	648.71	[79.17,96.33]
$\tilde{X}_2$	577.31	[59.17, 69.83]
$\tilde{X}_3$	788.18	[111.66, 133.84]

IT2 GFN-based allocations are displayed in Figure 4.

Tables 7, 8, and 9 present the allocations of Players 1, 2 and 3, respectively, across various α-cut levels (degree of confidence).

Figure 4

IT2 GFN-Based Allocations

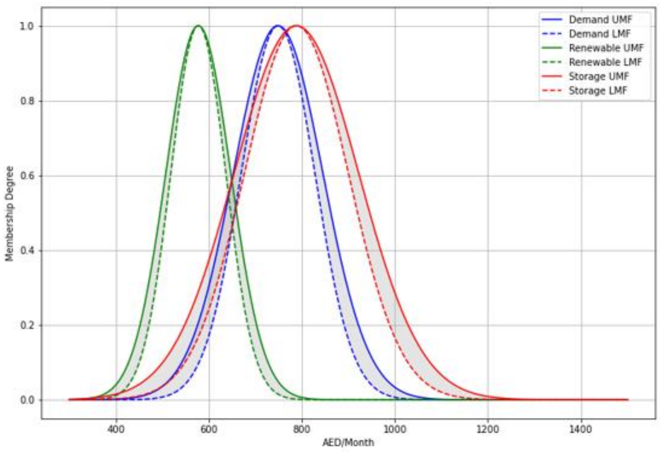


Table 7

α-Cut Interval of Player 1

α-Cut	$\underline{X}_1(\alpha) = [\overline{x}^l_1(\alpha), \underline{x}^l_1(\alpha)]$	$\overline{X}_1(\alpha) = [\underline{x}^r_1(\alpha), \overline{x}^r_1(\alpha)]$
1	648.71	648.71
0.5	[535.29, 555.49]	[741.93, 762.13]
0.1	[441.99, 478.81]	[818.61, 855.43]

Table 8

α-Cut Interval of Player 2

α-Cut	$\underline{X}_2(\alpha) = [\overline{x}^l_2(\alpha), \underline{x}^l_2(\alpha)]$	$\overline{X}_2(\alpha) = [\underline{x}^r_2(\alpha), \overline{x}^r_2(\alpha)]$
1	577.31	577.31
0.5	[495.09, 507.64]	[646.98, 659.53]
0.1	[427.46, 450.33]	[704.29, 727.16]

Table 9

α-Cut Interval of Player 3

α-Cut	$\underline{X}_3(\alpha) = [\overline{x}^l_3(\alpha), \underline{x}^l_3(\alpha)]$	$\overline{X}_3(\alpha) = [\overline{x}^l_3(\alpha), \underline{x}^l_3(\alpha)]$
1	788.18	788.18
0.5	[630.60, 656.71]	[919.65, 945.76]
0.1	[500.96, 548.56]	[1027.80, 1075.40]

Discussions

At α = 1.0 (complete confidence), the allocation for each player corresponds to the mean of the IT2-GFN. Player 1 (demand) saves 648.71 AED/month in energy costs, Player 2 (renewable) earns 577.31 AED/month in revenue, and Player 3 (storage) earns 788.18 AED/month in revenue. These values serve as reference points for decision-making under conditions of complete certainty. Compared to the corresponding coalition values in Table 3, all players benefit from cooperation. At α = 0.5 (moderate confidence), the allocations are expressed as intervals. Player 1 saves [535.29, 555.49] AED/month as a low (conservative) estimate and [741.93, 762.13] AED/month as a high (optimistic) estimate. Player 2's revenue ranges from [495.09, 507.64] AED/month as a low estimate to [646.98, 659.53] AED/month as a high estimate. Player 3 receives [630.60, 656.71] AED/month as a low estimate and [919.65, 945.76] AED/month as a high estimate.

Finally, at α = 0.1 (low confidence), Player 1 saves [441.99, 478.81] AED/month as a conservative estimate and [818.61, 855.43] AED/month as an optimistic estimate. Player 2's revenue ranges from [427.46, 450.33] AED/month as a low estimate to [704.29, 727.16] AED/month as a high estimate. Player 3 earns [500.96, 548.56] AED/month as a conservative estimate and [1027.80, 1075.40] AED/month as an optimistic estimate.

Comparative Analysis with the Interval Shapley Value

To highlight the advantages of the proposed method, the allocation strategy is reassessed using the interval-based Shapley value vector $\boldsymbol{\Phi}(\tilde{v})$, as described by Han et al. (2012).

The interval Shapley-like value $\Phi_i(\tilde{v})(\alpha)$ $(i = 1,2,3)$ is defined in Equation 50.

$$\Phi_i(\tilde{v})(\alpha) = \sum_{S\subseteq N/\{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot (\tilde{v}(S \cup \{i\})(\alpha) - \tilde{v}(S)(\alpha)) \tag{50}$$

For IT2-GFNs, $\Phi_i(\tilde{v})(\alpha) = \left[\underline{\Phi}_i(\alpha), \overline{\Phi}_i(\alpha)\right]$. Here, $\underline{\Phi}_i(\alpha)$ and $\overline{\Phi}_i(\alpha)$ are given by Equations 51 and 52 as follows:

$$\underline{\Phi}_i(\alpha) = \sum_{S\subseteq N/\{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot \left(\underline{v}(S \cup \{i\})(\alpha) - \underline{v}(S)(\alpha)\right), \tag{51}$$

and

$$\overline{\Phi}_i(\alpha) = \sum_{S\subseteq N/\{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot (\overline{v}(S \cup \{i\})(\alpha) - \overline{v}(S)(\alpha)). \tag{52}$$

For Player 1 $(i = 1)$, $\underline{\Phi}_i(\alpha)$ and $\overline{\Phi}_i(\alpha)$ are computed in Equations 53 and 54 as follows:

$$\underline{\Phi}_1(\alpha) = \sum_{S\subseteq N/\{1\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot \left(\underline{v}(S \cup \{1\})(\alpha) - \underline{v}(S)(\alpha)\right)$$

$$= \frac{1}{6}\left[2\left(\underline{v}(\{1\})(\alpha) - \underline{v}(\{\,\})(\alpha)\right) + \left(\underline{v}(\{1,2\})(\alpha) - \underline{v}(\{\,2\})(\alpha)\right) + \left(\underline{v}(\{1,3\})(\alpha) - \underline{v}(\{\,3\})(\alpha)\right) + 2\left(\underline{v}(N)(\alpha) - \underline{v}(\{2,3\,\})(\alpha)\right)\right]$$

$$= \left[648.71 - 100.83\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}, 648.71 - 74.67\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}\right] \tag{53}$$

and

$$\overline{\Phi}_1(\alpha) = \sum_{S\subseteq N/\{1\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot (\overline{v}(S \cup \{1\})(\alpha) - \overline{v}(S)(\alpha))$$

$$= \frac{1}{6}[2(\overline{v}(\{1\})(\alpha) - \overline{v}(\{\,\})(\alpha)) + (\overline{v}(\{1,2\})(\alpha) - \overline{v}(\{\,2\})(\alpha)) + (\overline{v}(\{1,3\})(\alpha) - \overline{v}(\{\,3\})(\alpha)) + 2(\overline{v}(N)(\alpha) - \overline{v}(\{2,3\,\})(\alpha))]$$

$$= \left[648.71 + 74.67\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}, 648.71 + 100.83\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}\right]. \tag{54}$$

Similarly, Equations 55 to 58 represent the computations of $\underline{\Phi}_i(\alpha)$ and $\overline{\Phi}_i(\alpha)$ for Player 2 where $i = 2$ and Player 3 where $i = 3$.

$$\underline{\Phi}_2(\alpha) = \sum_{S \subseteq N/\{2\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot \left(\underline{v}(S \cup \{2\})(\alpha) - \underline{v}(S)(\alpha)\right)$$

$$= \left[577.31 - 79.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}, 577.31 - 50.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}\right] \quad (55)$$

$$\overline{\Phi}_2(\alpha) = \sum_{S \subseteq N/\{2\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot \left(\overline{v}(S \cup \{2\})(\alpha) - \overline{v}(S)(\alpha)\right)$$

$$= \left[577.31 + 50.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}, 577.31 + 79.00\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}\right], \quad (56)$$

$$\underline{\Phi}_3(\alpha) = \sum_{S \subseteq N/\{3\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot \left(\underline{v}(S \cup \{3\})(\alpha) - \underline{v}(S)(\alpha)\right)$$

$$= \left[788.18 - 135.83\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}, 788.18 - 109.66\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}\right], \quad (57)$$

and

$$\overline{\Phi}_3(\alpha) = \sum_{S \subseteq N/\{3\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot \left(\overline{v}(S \cup \{3\})(\alpha) - \overline{v}(S)(\alpha)\right)$$

$$= \left[788.18 + 109.66\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}, 788.18 + 135.83\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}\right]. \quad (58)$$

Summing the lower intervals of the interval-based Shapley values for the three players yields Equation 59.

$$\underline{\Phi}_1(\alpha) + \underline{\Phi}_2(\alpha) + \underline{\Phi}_3(\alpha) =$$

$$\left[2014.20 - 314.83\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}, 2014.20 - 234.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}\right], \quad (59)$$

Conversely, summing the upper intervals of the interval-based Shapley values for the three players results in Equation 60.

$$\overline{\Phi}_1(\alpha) + \overline{\Phi}_2(\alpha) + \overline{\Phi}_3(\alpha) =$$

$$\left[2014.20 + 234.33\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}, 2014.20 + 315.66\sqrt{2\,ln\left(\frac{1}{\alpha}\right)}\right]. \quad (60)$$

Notice that, for all $\alpha \in (0,1)$:

$$\underline{\Phi}_1(\alpha) + \underline{\Phi}_2(\alpha) + \underline{\Phi}_3(\alpha) \neq \underline{v}(N)(\alpha)$$

and

$$\overline{\Phi}_1(\alpha) + \overline{\Phi}_2(\alpha) + \overline{\Phi}_3(\alpha) \neq \overline{v}(N)(\alpha).$$

Hence, $\boldsymbol{\Phi}(\tilde{v})(\alpha) \neq \tilde{v}(N)(\alpha)$, and the grand coalition value is not fully distributed among the players. In other words, the allocation proposed by Han et al. (2012) does not always guarantee an efficient distribution of the grand coalition value among its members for all $\alpha \in (0,1)$. In contrast, it can be verified that for all $\alpha \in (0,1]$:

$$\underline{X}_1(\alpha) + \underline{X}_2(\alpha) + \underline{X}_3(\alpha) = \underline{v}(N)(\alpha),$$

and

$$\overline{X}_1(\alpha) + \overline{X}_2(\alpha) + \overline{X}_3(\alpha) = \overline{v}(N)(\alpha).$$

Hence, $\widetilde{\boldsymbol{X}}(\boldsymbol{\alpha}) = v(N)(\alpha)$, indicating that the method proposed in this paper ensures a complete distribution of the grand coalition value among the players, thereby satisfying the efficiency condition.

Moreover, when comparing the width of the FOU, $\left|\left[\underline{\sigma}, \overline{\sigma}\right]\right|$, of the interval-based Shapley value derived using Moore's subtraction operator with that of the proposed IT2 GFN-based allocation, we observe that the former exhibits a wider FOU, indicating uncertainty magnification and, consequently, variance distortion.

Comparative Analysis with the Hukuhara Shapley Value

As defined by Yu and Zhang (2010), the Hukuhara Shapley value is represented by Equation 61:

$$\emptyset_i(\tilde{v})(\alpha) = \sum_{S \subseteq N/\{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot (\tilde{v}(S \cup \{i\})(\alpha) -_H \tilde{v}(S)(\alpha)), \tag{61}$$

where $(\tilde{v}(S \cup \{i\})(\alpha) -_H \tilde{v}(S)(\alpha)$ represents the Hukuhara difference between $(\tilde{v}(S \cup \{i\})(\alpha)$ and $\tilde{v}(S)(\alpha)$.

For IT2-GFNs, $\Phi_i(\tilde{v})(\alpha) = \left[\underline{\Phi}_i(\alpha), \overline{\Phi}_i(\alpha)\right]$. Here, $\underline{\Phi}_i(\alpha)$ and $\overline{\Phi}_i(\alpha)$ are given by Equations 62 and 63 as follows:

$$\underline{\Phi}_i(\alpha) = \sum_{S \subseteq N/\{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot \left(\underline{v}(S \cup \{i\})(\alpha) -_H \underline{v}(S)(\alpha)\right), \tag{62}$$

and

$$\overline{\Phi}_i(\alpha) = \sum_{S \subseteq N/\{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot (\overline{v}(S \cup \{i\})(\alpha) -_H \overline{v}(S)(\alpha)). \tag{63}$$

The existence of the Hukuhara Shapley value vector $\mathbf{\Phi}(\tilde{v})$ depends on the existence of the Hukuhara difference, which is overly restrictive. In the adopted IT2-GFN Hukuhara Shapley value, it exists only if the following conditions are satisfied for all $\forall\ i \in N$ and $\alpha \in (0,1]$ as in Equations 64 and 65.

$$\underline{v}^l(S \cup \{i\})(\alpha) - \overline{v}^l(S \cup \{i\})(\alpha) \geq \underline{v}^l(S)(\alpha) - \overline{v}^l(S) \quad \forall\ \alpha \in (0,1] \tag{64}$$

and

$$\overline{v}^r(S \cup \{i\})(\alpha) - \underline{v}^r(S \cup \{i\})(\alpha) \geq \overline{v}^r(S)(\alpha) - \underline{v}^r(S) \quad \forall\ \alpha \in (0,1]. \tag{65}$$

On the other hand, our proposed allocation method imposes no such restrictions, making it more broadly applicable.

CONCLUSION

This study extends fuzzy cooperative game theory by introducing an interval Type-2 Gaussian fuzzy game to address the increased uncertainty in coalition values resulting from the integration of EVs into DES. By representing coalition values and player allocations through IT2-GFNs, the proposed approach effectively models the higher levels of imprecision arising from uncoordinated EV charging and discharging patterns in DES. The proposed approach avoids Type-1 subtraction operations, which tend to amplify the FOU and can lead to unrealistic variance estimates or limit the model’s applicability. A comparative analysis with the interval-based Shapley value based on Moore’s subtraction operator demonstrates that the proposed method achieves allocation efficiency by fully distributing the grand coalition value among players while overcoming the known limitations of the Hukuhara difference. Future research could explore the axiomatic characterisation of the proposed allocation to further establish its theoretical fairness properties.

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