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# Expert-Elicited Fuzzy Membership Functions for Emergency Information System

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## ABSTRACT

In emergency information systems, expert decision-making is a critical process that often employs linguistic terms to convey subjective judgments. Membership functions (MFs) are an essential tool for representing the meaning of these terms, which allows them to be processed computationally. Existing methods for eliciting MFs of these terms struggle to capture the inherent variability and probabilistic nature of expert opinions, hindering accurate representation of uncertainty. This study proposes a framework for eliciting MFs of probabilistic linguistic terms using the Interval Estimation (IE) technique. The procedure integrates a graphic survey to construct MFs that fulfil the definition of Triangular Fuzzy Number (TFN). This approach enables experts to directly express their uncertainty ranges, resulting in a high level of consistency and thereby validating the shared and context-dependent understanding. A case study on eliciting MFs and integrating them with a Bayesian Network (BN) decision model for emergency evacuation, demonstrating the utility of our proposed framework. The results indicate that an evacuee's ability to perceive fire cues and hazardous events, while maintaining psychological stability, increases the probability of a successful evacuation. The performance analysis, as indicated by sensitivity values, confirms the stability and robustness of the BN model parameters, thereby validating the rationality and meaningfulness of the MFs. The resulting MFs demonstrated a significant improvement in capturing the uncertainty of expert assessment in the emergency information system compared to traditional methods.

**Keywords:** Decision-support systems, emergency information systems, expert decision-making, fuzzy linguistic, membership functions.

## INTRODUCTION

Emergencies, especially during evacuations, are inherently complex and uncertain, often involving rapidly changing conditions, incomplete information, and high stakes for both responders and affected populations. In such high-pressure situations, the role of decision-support systems (DSS) in emergency information systems has become increasingly vital. DSS can significantly enhance the decision-making process by integrating expert input, optimising evacuation strategies, and providing real-time recommendations based on the evolving emergency context. The involvement of experts in DSS is crucial, as their domain knowledge, experience, and judgment help fill the gaps when data is incomplete or ambiguous. Experts contribute valuable insights that improve the accuracy and relevance of the system's recommendations, ensuring that decisions are not solely based on automated algorithms but also on human intuition and expertise (Jing & Tang, 2014; Miller-Hooks & Krauthammer, 2007). Studies show that expert-driven decision-making contributes to more efficient and safer evacuation outcomes during emergencies (Li et al., 2021; Adesina et al., 2022; Su et al., 2022).

Expert decision-making (EDM) processes often involve navigating complex uncertainties, particularly in situations where information is incomplete or ambiguous, as is often the case in emergency scenarios. In such scenarios, individuals naturally rely on imprecise linguistic judgments, often preferring qualitative assessments over precise numerical probabilities. Linguistic expressions such as "likely," "thinkable," or "expected" provide an intuitive means of capturing uncertainty, surpassing the rigidity of numerical values. Linguistic terms are indispensable for expressing subjective probabilities and reflecting human reasoning patterns (Wallsten et al., 1993).. This preference for linguistic descriptions necessitates robust methods to manage the inherent ambiguity in human decision-making. While intuitive for communication, these expressions pose challenges for computational modelling, driving researchers to develop techniques for translating them into numerical equivalents. However, such translations inevitably introduce uncertainties. To mitigate this, fuzzy set theory has emerged as a valuable tool for modelling vague and imprecise information (Zadeh, 1965). Fuzzy logic, through fuzzy membership functions (MFs), enables the representation of uncertainty and vagueness by mapping linguistic terms onto numerical scales (Bocklisch et al., 2010). This mapping facilitates a systematic interpretation of verbal probability expressions, converting subjective judgments into quantifiable values and thus, precisely quantifying subjective uncertainty (Bocklisch et al., 2010; Rodríguez et al., 2016).

The translation of linguistic terms into MFs is often based on predefined scales or assumptions made by researchers. In contrast, scales such as those proposed by Chen and Hwang (1992) have been widely used; however, their generalisation limits their applicability in field-specific contexts where domain expertise is critical. It is a significant challenge because the very representation of linguistic terms shapes human reasoning and supports decision-makers in various fields, such as medicine, business, and risk management (Bocklisch et al., 2010). A single, universal scale often fails to capture the unique nuances of different disciplines. The fact that there is no universal distribution of concepts makes it difficult to get agreement on the same MFs for linguistic terms (Herrera & Herrera-Viedma, 2000). For instance, a conversion scale for medical assessments may differ significantly from one used for climate indicators, as each domain has unique data requirements and validation processes. It is crucial to ensure that a term or expression accurately conveys information within its specific context (Silva & Morais, 2014).

Studies that customise fuzzy linguistic scales for specific domains in emergency management demonstrate enhanced accuracy and efficacy in expert assessments. These studies involve expert input in terms of linguistic values in areas such as evaluating emergency response capacity (Ju et al., 2012), general emergency decision-making (Ding et al., 2020b; Liu et al., 2024; Mo, 2020; Wang et al., 2017), group decision-making for emergency responses (Ding et al., 2020a; Shen et al., 2025; Sun et al., 2021) as well as specific applications such as natural disaster management systems (Adesina et al., 2022), waterlogging disaster management in subway stations (Wu et al., 2020), emergency preparedness against flood risk (Amin et al., 2019), off-shore safety assessement (Ren et al., 2008; 2009), and meteorological preparedness (Santacreu Rios et al., 2015; Yan et al., 2025). While these studies highlight the value of customised fuzzy scales in their respective domains, they often rely on predefined, data-driven approaches or time-consuming manual interviews. It leaves a significant gap in the field, which still lacks a formal and efficient expert-elicitation process, particularly in emergency information systems.

Therefore, the objective of this study is to propose a framework for the fuzzification of verbal probability expressions through expert elicitation to enhance the practical use of linguistic scales in decision-making problems for emergency information systems. The methodology involves designing a graphic survey to engage domain experts in defining and calibrating MFs that comply with the principles of fuzzy numbers. This approach provides a structured and context-dependent elicitation of fuzzy MFs, leading to a more accurate and reliable representation of uncertainty in decision-making.

## RELATED WORKS

### Expert Decision-Making in Emergency Information Systems

The process of EDM in emergencies requires rapid, accurate, and consensus-driven actions under immense pressure and uncertainty. This process is significantly enhanced by advanced analytical decision-support tools that enable experts to evaluate complex scenarios systematically. A wide range of mathematical methods are used for this, including Bayesian Networks (BN), game theory, Markov decision processes, fuzzy set theory, and various hybrid approaches. For instance, BNs are highly effective for probabilistic reasoning and risk assessment in emergencies (Hao et al., 2018; Lang et al., 2020; Qiu et al., 2016; Xue et al., 2021). They function by modelling the dependencies between different variables, which allows decision-makers to update their risk assessments as new information becomes available. Meanwhile, game theory is a valuable tool for situations involving adversarial decision-making and resource allocation, especially when multiple parties with conflicting interests are involved (Chen et al., 2023; Ding & Liu, 2019). It models the strategic interactions between agents, such as different emergency response teams or government agencies, to help anticipate the actions of others and identify optimal strategies under uncertainty. For dynamic, time-evolving emergency scenarios, Markov decision processes (MDPs) provide a robust framework (Gao et al., 2023). They help decision-makers determine the best sequence of actions over time, with each decision impacting the future state of the system.

The fuzzy set theory is a cornerstone of emergency decision-making because it excels at handling the vagueness and imprecision that are common in these situations (e.g. Liu et al., 2024; Ren et al., 2017; Rong et al., 2021; Sun et al., 2015, 2021; Yan et al., 2025). Instead of traditional true/false logic, it allows for degrees of membership, making it ideal for incorporating qualitative data and expert opinions expressed in linguistic terms (Mohd Sharif et al., 2019). In multi-attribute group decision-making, extensions such as hesitant, intuitionistic, and picture fuzzy sets support the process of building consensus and ranking potential emergency plans, which is essential for integrating subjective input from various experts and leading to more robust decisions. Hybrid approaches, which combine the strengths of various analytical methods, are frequently employed for complex modelling. For example, a hybrid fuzzy-Bayesian Network can leverage the probabilistic and causal modelling of BNs to understand a scenario's likelihoods while simultaneously using fuzzy logic to handle the imprecise and subjective data provided by field experts (Hao et al., 2018; Ramli et al., 2021; Ren et al., 2009). In a hybrid AI-fuzzy approach, AI methods like machine learning (ML) can be combined with fuzzy logic to address uncertainty. For example, a study by Tariq et al. (2020) combined a recursive partitioning and regression decision tree (RPART) with a rule-based fuzzy system to address the vagueness and uncertainty inherent in expert decisions during non-recurrent congestion of traffic incidents at intersections. This combination results in a more comprehensive and resilient analytical framework that can accommodate a wide range of uncertainties, from objective data to subjective human judgments.

**Fuzzy Logic and Membership Functions**

A fuzzy MF quantifies the degree to which a given input belongs to these linguistic categories, enabling the handling of uncertainty and imprecision in decision-making processes (Zadeh, 1965). Fuzzy sets differ from classical sets, where classical sets explain the precise (crisp) properties of membership while fuzzy sets explain the imprecise properties of membership of objects. In other words, the membership of an object in a fuzzy set can be approximate.

A fuzzy set of $\tilde{A}$ in universe of discourse $X$ is defined as $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | x \in X\}$ where $\mu_{\tilde{A}}(x)$ is the MF of $\tilde{A}$ which maps each element $x$ in $X$ to a real interval [0,1] (Zadeh, 1965). A fuzzy set of satisfy the following properties of normality, convexity, fuzzy set and overlapping (Dubois & Prade, 1980; Klir & Yuan, 1995; Ross, 2010; Zadeh, 1965):

(a) A fuzzy set $\tilde{A}$ is called normal when the height of $\tilde{A}$ is equal to 1 or having a maximum membership value of 1, otherwise it is called subnormal.
(b) For any elements $a$, $b$ and $c$ in a fuzzy set $\tilde{A}$, the relation $a < b < c$ implies that $\mu_{\tilde{A}}(b) \geq \min[\mu_{\tilde{A}}(a), \mu_{\tilde{A}}(c)]$, then $\tilde{A}$ is said to be a convex fuzzy set, otherwise it is called nonconvex.
(c) A fuzzy number is a fuzzy set $\tilde{A}$ on the real line $\mathbb{R}$ having the properties of normal and convex.
(d) Two fuzzy sets, $\tilde{A}$ and $\tilde{B}$ are overlapping each other if $\tilde{A} \cap \tilde{B} \neq \emptyset$.

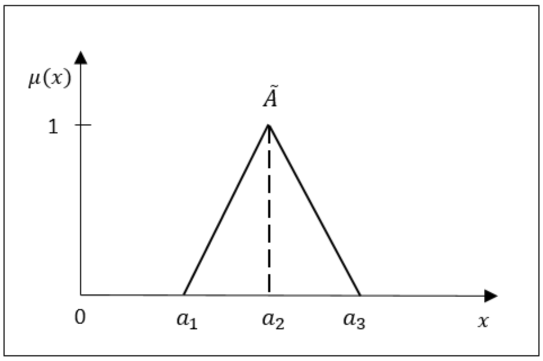
In this study, Triangular Fuzzy Numbers (TFNs) which are parameterised by triplet numbers will be used because they are widely preferred in fuzzy set theory due to their simplicity, computational efficiency, and ability to effectively represent linguistic variables, making them practical for reducing complexity and facilitating decision-making in uncertain environments. A positive TFN, $\tilde{A}$ can be defined as $(a_1, a_2, a_3)$, where the MF $\mu_{\tilde{A}}(x)$ is defined in Equation 1.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & x > a_3 \end{cases} \tag{1}$$

where $a_1$, $a_2$ and $a_3$ are the parameters of the fuzzy number. The parameters $a_1$ and $a_3$ are respectively the lower least likely value and upper least likely value, whereas $a_2$ is the most likely value, or middle value of the fuzzy number. The point $a_2$ with membership degree of 1 is called the mean value, whereas $a_2 - a_1$ and $a_3 - a_2$ are the left and right spreads of $a_2$, respectively. Generally, the shape of the TFN can be either symmetric (both its spreads are equal) or asymmetric (both its spreads are unequal). The graphical shape of the symmetric TFN is presented in Figure 1.

**Figure 1**

*Graphical Presentation of a TFN*




**Expert Elicitation of Fuzzy Membership Functions**

Fuzzification is the process of transforming a crisp, precise value into a fuzzy one. The need for this process arises from the understanding that many quantities that are assumed to be certain actually carry inherent uncertainty. When this uncertainty stems from imprecision, ambiguity, or vagueness, the variable is considered fuzzy and can be represented by an MF (Ross, 2000). The development of fuzzy MFs, or the process of fuzzification, is influenced by several factors, including the characteristics of the data, the type of fuzzy set employed, and the specific context in which the system operates. While various methods exist, such as data-driven approaches, the development often relies on expert elicitation, where domain knowledge and human judgment are used to define the fuzzy sets.

Eliciting fuzzy MFs through expert input is crucial when objective data is lacking or ambiguous, especially in decision-making and modelling tasks. This process involves translating expert knowledge or preferences into mathematical representations that define the degree of membership of elements within fuzzy sets. Several approaches to expert elicitation for assigning MFs to linguistic variables have been identified in the literature. These include group expert assessment, user- and data-driven methods, and direct expert data methods. The group expert assessment aggregates individual expert judgments using advanced mathematical frameworks to synthesise group decisions. For example, Interval Type-2 fuzzy sets can be used to represent uncertainty by capturing a range of possible membership values when experts disagree, rather than relying on a single function (Pagola et al., 2013). Meanwhile, user- and data-driven methods combine both expert input and data analysis approaches. For example, the Analytic Hierarchy Process (AHP) integrates statistical data analysis with expert evaluation to structure and prioritise criteria for selecting and shaping MFs (Shushura, 2020). Automated methods, such as neural networks and fuzzy clustering, refine MFs based on expert input and data patterns (Xu et al., 2023).

The direct expert data methods, also known as manual methods, use mathematical data approximation to construct MFs from expert-provided values (Guliyeva & Ismibayli, 2023). These methods are often favoured for their ability to capture the complexities of specialised decision-making environments. It involves humans' contextual and semantic knowledge about an issue, typically by experts in deriving the MFs (Ross, 2010). As mentioned by Dubois and Prade (1980), MFs are the tendency values that are subjectively assigned by humans and are context dependent. A number of methods are available to elicit MFs from expert knowledge as found in the literature: Polling (P), Point Estimation (PE), Interval Estimation (IE), Reverse Rating (RR), Membership Function Exemplification (MFE), Pairwise Comparison (PC), and Transition Interval Estimation (TIE) among others (Chameau & Santamarina, 1987; Klir & Yuan, 2015; Turksen, 1991). Unlike data-driven methods, expert elicitation using direct expert data approaches leverages domain-specific knowledge to account for contextual nuances, ensuring tailored solutions for complex systems. This process enables experts to tailor MFs to accurately reflect real-world complexities, making them particularly valuable in high-stakes environments where decisions must be robust and justifiable.

Despite its benefits, expert elicitation using direct expert data in the construction of fuzzy MFs faces several challenges. Subjectivity and potential bias introduced by experts can affect the reliability of elicited MFs (Miliauskaitė & Kalibatiene, 2020). Bias may arise from personal experiences, cognitive limitations, or the framing of elicitation tasks (O'Hagan, 2019). Consistency in expert input can also be difficult to achieve, particularly when multiple experts are involved, as differing interpretations or opinions may emerge (Herrera & Herrera-Viedma, 2000). Moreover, the lack of standardised frameworks across domains further complicates ensuring repeatability and comparability of results (Ross, 2010). Addressing these issues requires the development of structured elicitation methods, better training of experts, and the implementation of bias-mitigation strategies to improve the robustness and reliability of expert-derived MFs.

As far as the study is concerned, the development of fuzzy MFs for emergency information systems is an underexplored area. This study addresses that gap by systematically customising fuzzy linguistic scales for EDM in emergency information systems. Our approach utilises the IE method, combined with a graphic survey, to capture the full range of expert opinions on probabilistic linguistic terms. This approach enhances existing methods by capturing expert variability and providing more context-sensitive MFs, which are essential for informed decision-making in critical domains. The next section outlines the methodology employed in this study.

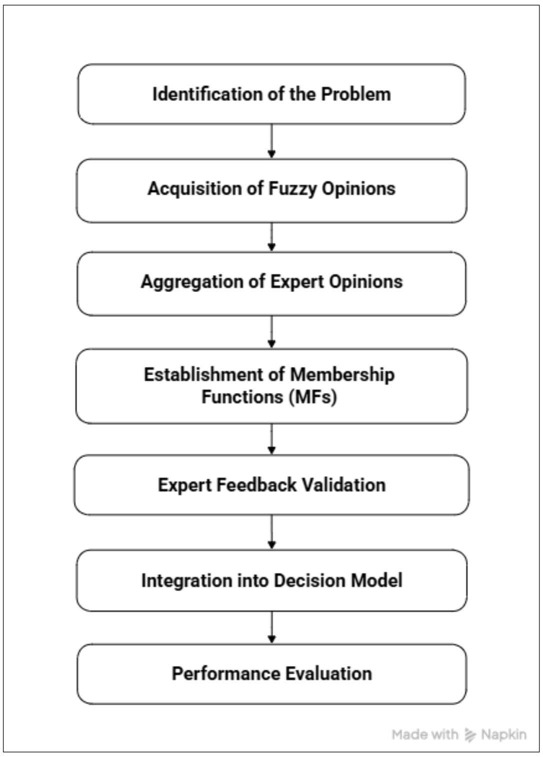
## METHODOLOGY

### Overview of the Proposed Framework

In this section, we present a methodological framework designed for the acquisition and aggregation of expert opinions in the construction of fuzzy MFs for EDM in an emergency information system. This framework aims to address the inherent challenges of subjective uncertainty by providing a structured approach to expert elicitation, as illustrated in the comprehensive process flow outlined in Figure 2.

**Figure 2**

*Process Flow for Expert Elicitation of MFs*




To provide a clear and structured proposed framework, the detailed steps are as follows:

(a) Identification of the problem - The initial phase involves precisely defining the problem. This necessitates a thorough understanding of the context wherein fuzzy logic and linguistic terms are employed. The objectives of MF development are articulated, and experts with pertinent knowledge are selected.
(b) Acquisition of fuzzy opinions using a graphic survey - Expert opinions are obtained through a graphic survey utilising the IE technique. Experts provide a range of values for linguistic terms, facilitated by graphical aids like sliders or scales.
(c) Aggregation of expert opinions on MFs - Upon data collection, aggregation procedures synthesise opinions into unified MFs. This phase includes mathematical aggregation techniques, with potential weighting based on expert confidence.
(d) Establishment of MFs of linguistic terms - Following aggregation, MFs are refined and established. Verification ensures adherence to the characteristics of fuzzy logic, including fuzzy number properties, shape, overlap, and vagueness. This validation of MFs also includes a consistency analysis of MFs among experts.
(e) Expert feedback validation - Contextual relevance is evaluated through expert review, ensuring MFs accurately reflect expert knowledge and domain-specific context.
(f) Integration into decision model - Finalised MFs are integrated into applicable decision models, facilitating the practical application of expert knowledge.
(g) Performance evaluation - The performance of the decision model incorporating MFs is evaluated, including sensitivity analysis, to ensure ongoing effectiveness and relevance.

The following section presents the application of the proposed framework to a problem in EDM in an emergency information system.

## CASE STUDY

### Expert Elicitation of Membership Functions for Bayesian Network Evacuation Modelling

This study presents a systematic procedure for eliciting MFs from experts, which are then integrated into a BN decision model designed for emergency evacuation scenarios. To demonstrate the application of this procedure, the steps in the proposed framework are elaborated in the following sections.

### Identification of the Problem

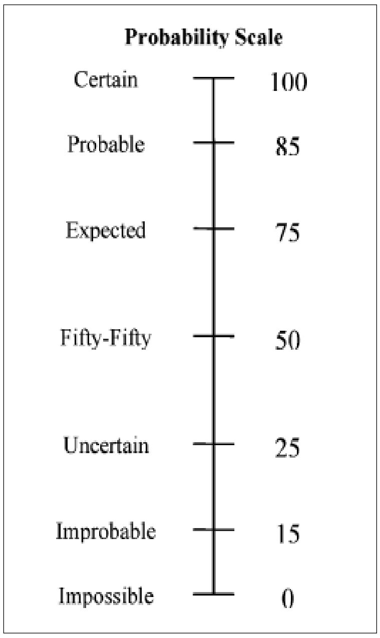
Responses during fire evacuations are inherently complex and influenced by various factors, including an individual's perception of risk, emotional states, and cognitive abilities (Ramli et al., 2021). These factors exhibit significant subjectivity and variability, making them challenging to quantify with traditional numerical scales. Furthermore, these responses are highly context-dependent and dynamic, influenced by factors like fire visibility, alarms, and crowd density, highlighting the inadequacy of generalised scales. Instead, a context-specific linguistic scale informed by expert input is necessary. For this study, seven experts from academia and professional fire practitioners participated.

### Acquisition of Fuzzy Opinions using a Graphic Survey

One well-known assessment tool used in the expert elicitation decision-making process is the numerical probability scale, which contains seven probabilistic linguistic terms: Impossible (IPS), Improbable (IPR), Uncertain (UNC), Fair-Chance (FCH), Expected (EXP), Probable (PRO) and Certain (CER) developed by Renooij and Witteman (1999) as part of an elicitation method for BN. This scale is illustrated in Figure 3, where a horizontal or vertical line is divided into non-overlapping intervals, with verbal anchors assigned to corresponding numerical values (0–100% or 0–1). To define these terms, experts were asked to provide interval values for each of the seven terms through a graphic survey. The meaning of these terms is then represented as a TFN on a scale of [0, 100], which was used to guide the expert in eliciting the interval.

**Figure 3**

*Probability Scale by Renooij and Witteman (1999)*

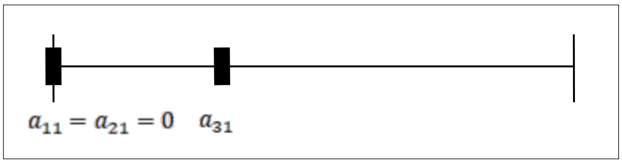



The procedure for acquiring and processing linguistic labels used by experts involves an expert-elicitation process through the IE method, accompanied by a graphic survey. A similar graphic survey approach for developing MFs was previously employed by García-Cascales and Lamata (2007) for multi-criteria decision-making (MCDM) problems in manufacturing, which serves as a baseline for the present study. The detailed steps are outlined below.

(a) Step 1- Make the seven verbal probabilistic terms as linguistic terms.
(b) Step 2- Set the numerical crisp value of each linguistic term available in the probability scale as the middle value or the most likely value, $a_{2j}$ of TFN.
(c) Step 3- The first linguistic term, 'Impossible (IPS)' has the TFN parameter $(a_{11}, a_{21}, a_{31})$. The minimum and the middle values are set to be 0, $a_{11} = a_{21} = 0$. The expert is asked to mark the upper least likely value that is more than (or equal to) the middle value, $a_{31} \geq a_{21}$ (see Figure 4).

**Figure 4**

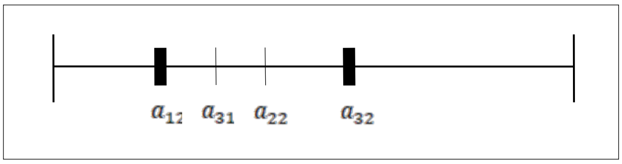
*Graphic Representation of Eliciting $a_{31}$*



(d) Step 4 - For the second linguistic term, 'Improbable (IPR)', the middle value is set to be $a_{22} = 15$. The expert is asked to mark the lower least likely value that is less than (or equal to) the middle value, $a_{12} \leq a_{22}$ and less than (or equal to) the upper least likely value of the first linguistic term 'Impossible (IPS)', $a_{31}$, i.e., $a_{12} \leq a_{31}$. Then, the expert is asked to mark the upper least likely value that is more than (or equal to) the middle value, $a_{32} \geq a_{22}$(see Figure 5).

**Figure 5**

*Graphic Representation of Eliciting $a_{12}$ and $a_{32}$*

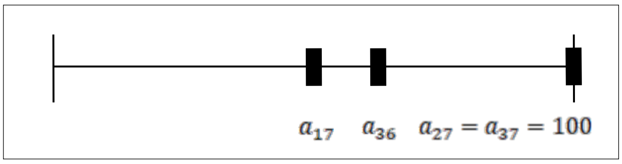



(e) Step 5 - Repeat Step 4 for other linguistic terms 'Uncertain (UNC)', 'Fair-chance (FCH)', 'Expected (EXP)' and 'Probable (PRO)'.

(f) Step 6 - For the last linguistic term, 'Certain (CER)', the middle and the upper least likely values are set to be 100, $a_{27} = a_{37} = 100$. The expert is asked to mark the lower least likely value that is less than (or equal to) the middle value, $a_{17} \leq a_{27}$ and less than (or equal to) the upper least likely value of 'Probable (PRO)', $a_{36}$, i.e., $a_{17} \leq a_{36}$ (see Figure 6).

**Figure 6**

*Graphic Representation of Eliciting $a_{17}$*




(g) Step 7: Obtain the fuzzy number corresponding to every linguistic term $j$ defined; where the support of the fuzzy number corresponds with the interval defined by the expert $(a_{1j}, a_{3j})$ and the central value of the fuzzy number is taken from the middle value, $a_{2j}$.

(h) Step 8: Normalised the fuzzy number by $\frac{a_{ij}}{100}$ in order to fulfill the interval of MFs of fuzzy number [0,1].

(i) Step 9: Finalise the Fuzzy Probability Scale where the linguistic terms and their corresponding TFNs is compiled into a complete scale, ready for applications in decision-making under uncertainty.

A series of test runs has been conducted with two domain academic experts to ensure that they understand the questions and answer them correctly. In this process, the graphic survey has been evaluated and revised accordingly before distributing the real elicitation. The elaboration of the graphic helps experts fully understand the elicitation procedure and avoid different interpretations and potential mistakes that could be made. Throughout the session, they were kept reminded that the interval values of the linguistic terms provided were based on their subjective judgment and closely related to the specific context of the EDM for an emergency evacuation event. They also had to ensure that the interval values provided fulfilled the definition of fuzzy numbers, overlapped, and were within an acceptable range. The graphic survey is designed in such a way that it can capture the numerical meanings of the linguistic terms and, at the same time, fulfil the conditions of TFN.

**Aggregation of Expert Opinions on Membership Functions**

A number of aggregation approaches have been applied in studies of fuzzy opinions, but are not limited to: aggregation operators-based techniques such as the weighted mean, ordered weighted averaging, max and min operators, entropy-based aggregation, Dempster Shafer's combination rule, Consistency Aggregation Method (CAM) and Similarity Aggregation Method (SAM). This study adopts the SAM method proposed by Hsu and Chen (1996) for aggregating fuzzy opinions due to its practical advantages, including the use of fuzzy set theory and its properties in the aggregation process, simplifying the mathematical analysis involved in group decision-making, and accounting for both the aggregation's uncertainty and the individual uncertainties of all experts. The detailed algorithm of the modified SAM aggregation procedure described by Deng and Shi (2003) is as follows:

(a) Each expert $E_u (u = 1, 2, \ldots, m)$ expresses an opinion by a set of fuzzy number, $\tilde{A}_u$ with MF of $\mu_{\tilde{A}}(x)$ to represent the estimate for the variable of interest *X*.

(b) Calculate the agreement degree or similarity measure function, $S(\tilde{A}_u, \tilde{A}_v)$ where $u, v = 1, 2, \ldots, m$ and $u \neq v$. The $S(\tilde{A}_u, \tilde{A}_v)$ refers to the pairwise similarity of the fuzzy opinions $\tilde{A}_u$ and $\tilde{A}_v$ between each pair of expert $E_u$ and expert $E_v$ using Equation 2.

$$S(\tilde{A}_u, \tilde{A}_v) = \frac{\int_x \left(\min\{\mu_{\tilde{A}_u}(x), \mu_{\tilde{A}_v}(x)\}\right) dx}{\int_x \left(\max\{\mu_{\tilde{A}_u}(x), \mu_{\tilde{A}_v}(x)\}\right) dx}. \tag{2}$$

If two experts have the same estimates, then $S(\tilde{A}_u, \tilde{A}_v) = 1$. The higher the percentage of overlap, the higher the agreement degree.

(c) Calculate the average agreement degree $AA(E_u)$ using Equation 3.

$$AA(E_u) = \frac{1}{m-1} \sum_{\substack{v=1 \\ v \neq u}}^{m} S(\tilde{A}_u, \tilde{A}_v), \tag{3}$$

(d) Calculate the relative agreement degree. $RAD(E_u)$ using Equation 4.

$$RAD(E_u) = \frac{AA(E_u)}{\sum_{u=1}^{m} AA(E_u)}, \tag{4}$$

(e) Calculate the degree of importance $w_u$ using Equation 5.

$$w_u = \frac{m_u}{\sum_{u=1}^{m} m_u}, \tag{5}$$

where $m_u$ is the relative importance of experts. In this study, we consider the expert's weight depending on their different expertise background which are discussed in the following section.

(f) Calculate the average weight agreement degree $AWAD(E_u)$ using $w_v$ and $S(\tilde{A}_u, \tilde{A}_v)$ where $w_v$e is the degree of importance of expert $E_v$ using Equation 6.

$$AWAD(E_u) = \frac{1}{m-1} \sum_{\substack{v=1 \\ v \neq u}}^{m} w_v S(\tilde{A}_u, \tilde{A}_v) . \tag{6}$$

(g) Calculate the relative weight agreement degree $RWAD(E_u)$ using Equation 7.

$$RWAD(E_u) = \frac{AWAD(E_u)}{\sum_{u=1}^{m} AWAD(E_u)} . \tag{7}$$

The relative weight agreement degree of expert denotes the consistency degree between the opinion of an expert $E_u$ with the more important expert.

(h) Define the consensus degree coefficient $CDC(E_u)$ using Equation 8.

$$CDC(E_u) = \beta_1 . w_u + \beta_2 . RAD(E_u) + \beta_3 . RWAD(E_u) \tag{8}$$

where $0 \leq \beta_i \leq 1$. $\beta_1$, $\beta_2$ and $\beta_3$ are the relaxation factors that are attached to the weight of the expert $w_u$ , the relative agreement degree $RAD(E_u)$ , and relative weight agreement degree $RWAD(E_u)$ respectively which refers to the importance of each measurement towards one another and can be determined subjectively by the decision maker. In this study, we set $\beta_1 > \beta_2, \beta_3$ to put more emphasis on the expert's weight compared to the rest of the measurements.

(i) Finally aggregate the fuzzy opinions $\tilde{A}$ of expert $E_u(u = 1, 2, \ldots, m)$ using Equation 9.

$$\tilde{A} = \sum_{u=1}^{m} \left(CDC(E_u)(.)\tilde{A}_u\right) . \tag{9}$$

**Formulation of Expert Weights**

In practice, experts with a higher quality or classification of their expertise are given a higher weight. To determine the weightings of the experts, it is possible to assign a score to each level of expertise classification. In this study, the SAM procedure is incorporated to weight experts based on their expertise background, resulting in a more robust and reliable aggregation of their opinions. Table 1 shows the scores that are assigned by ranking from the lowest to the highest level of classification for each constitution. In this study, we define the determination of the expert's weighting factor as $WS_u$ $WS_u$ the weighting score used to represent the quality of the different experts based on their constitutions. It is calculated by the total of classification of each constitution for the expert. Using $WS_u$ $WS_u$ , the new weighting factor, $w_u^*$ for each expert $E_u(u = 1, 2, \ldots, m)$ $E_u(u = 1, 2, \ldots, m)$ is determined as in Equation 10.

$$w_u^* = \frac{WS_u}{\sum_u^m WS_u} . \tag{10}$$

In this case, the weighting score for expert $u$, $WS_u WS_u$ = professional title score + area of expertise score + experience score + highest education level score + age score. The determination of the weighting score and the weighting factor for all experts is as in Table 2.

**Table 1**

*Score for Classification and Constitution of Different Expert*

| Constitution | Classification | Score |
|---|---|---|
| Professional title/ position | Professor | 4 |
| | Associate professor | 3 |
| | Senior lecturer/senior officer | 2 |
| | Lecturer/Officer | 1 |
| Area of expertise | Psychology/Fire | 2 |
| | Others (related) | 1 |
| Highest education level | Phd | 5 |
| | Masters | 4 |
| | Bachelor | 3 |
| | Diploma | 2 |
| | Certificate | 1 |
| Age | Greater than 50 years old | 4 |
| | 40-49 years old | 3 |
| | 30-39 years old | 2 |
| | Less than 30 years old | 1 |

**Table 2**

*Expert's Weighting Score and Weighting Factor*

| Expert | Professional Title/ Position | Area of Expertise | Experience | Highest Education Level | Age | $WS_u$ | $w_u^*$ |
|---|---|---|---|---|---|---|---|
| 1 | Associate professor | Psychology | > 21 years | Phd | 40-49 years old | 18 | 0.1875 |
| 2 | Senior lecturer | Psychology | 16-20 years | Phd | 40-49 years old | 16 | 0.1667 |
| 3 | Associate professor | Psychology | 16-20 years | Phd | 40-49 years old | 17 | 0.1771 |
| 4 | Senior lecturer | Psychology | 6-10 years | Phd | 30-39 years old | 13 | 0.1354 |
| 5 | Officer | Fire Practitioner | 11-15 years | Diploma | 40-49 years old | 12 | 0.125 |
| 6 | Officer | Fire Practitioner | 6-10 years | Diploma | 30-39 years old | 10 | 0.1042 |
| 7 | Officer | Fire Practitioner | 6-10 years | Diploma | 30-39 years old | 10 | 0.1042 |

The expert's weighting score and weighting factor (see Table 2) will be used to calculate the expert's weight, $w_u^*$ as shown in Equation 10. Meanwhile for calculating the $CDC^*$ in Equation 8, the relaxation factors are set as $\beta_1 = 0.5$, $\beta_2 = 0.3$ and $\beta_3 = 0.2$ after considering the degree of importance or weighting factor $w_u^*$ is more important than the relative agreement degree, $RAD$ and relative weight agreement degree, $RWAD$ as in Deng and Shi (2003). As an example, the detailed aggregation calculations for the maximum likely value, $a_{31}$ of the linguistic term 'Impossible (IPS)' are given in Table 3. In this table, the opinions from seven experts $E_u$ $(u = 1, 2, \ldots, 7)$ is denoted as $\tilde{A}_u$. Using all the steps provided in previous section, the aggregated value of expert opinions, $\tilde{A}^*$ is calculated.

**Table 3**

*Aggregation Calculation for $a_{31}$ of 'Impossible (IPS)'*

| Expert, $u$ | $\tilde{A}_u$ | $AA_u$ | $RAD_u$ | $w_u^*$ | ${AWAD_u}^*$ | ${RWAD_u}^*$ | ${CDC_u}^*$ | $\tilde{A}^*$ |
|---|---|---|---|---|---|---|---|---|
| E1 | 5 | 0.962 | 0.142 | 0.188 | 0.131 | 0.135 | 0.163 | 0.13 |
| E2 | 20 | 0.974 | 0.144 | 0.167 | 0.134 | 0.139 | 0.154 | |
| E3 | 0 | 0.948 | 0.140 | 0.177 | 0.130 | 0.135 | 0.158 | |
| E4 | 10 | 0.971 | 0.143 | 0.135 | 0.140 | 0.145 | 0.140 | |
| E5 | 18 | 0.975 | 0.144 | 0.125 | 0.142 | 0.147 | 0.135 | |
| E6 | 20 | 0.974 | 0.144 | 0.104 | 0.145 | 0.150 | 0.125 | |
| E7 | 20 | 0.974 | 0.144 | 0.104 | 0.145 | 0.150 | 0.125 | |
| $S(\hat{A}_u, \hat{A}_v)$ | | | | | | | | |
| S(E1,E2) | 0.950 | S(E2,E4) | 0.967 | S(E3,E7) | 0.933 | | | |
| S(E1,E3) | 0.983 | S(E2,E5) | 0.993 | S(E4,E5) | 0.973 | | | |
| S(E1,E4) | 0.983 | S(E2,E6) | 1.000 | S(E4,E6) | 0.967 | | | |
| S(E1,E5) | 0.957 | S(E2,E7) | 1.000 | S(E4,E7) | 0.967 | | | |
| S(E1,E6) | 0.950 | S(E3,E4) | 0.967 | S(E5,E6) | 0.993 | | | |
| S(E1,E7) | 0.950 | S(E3,E5) | 0.940 | S(E5,E7) | 0.993 | | | |
| S(E2,E3) | 0.933 | S(E3,E6) | 0.933 | S(E6,E7) | 1.000 | | | |

The results of the aggregated TFN of each linguistic term are presented in Table 4 and displayed graphically in Figure 7. We now define the linguistic terms with their MFs as the Fuzzy Probability Scale.

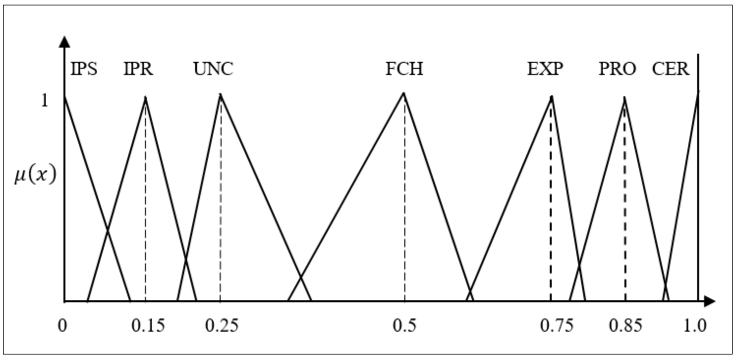
**Table 4**

*TFN of Fuzzy Probability Scale*

| Linguistic Terms | TFN |
|---|---|
| Impossible (IPS) | (0, 0, 0.13) |
| Improbable (IPR) | (0.07, 0.15, 0.22) |
| Uncertain (UNC) | (0.19, 0.25, 0.37) |
| Fair-chance (FCH) | (0.34, 0.50, 0.59) |
| Expected (EXP) | (0.58, 0.75, 0.78) |
| Probable (PRO) | (0.77, 0.85, 0.92) |
| Certain (CER) | (0.90, 1.0, 1.0) |

**Figure 7**

*Graphical TFN of Fuzzy Probability Scale*




**Establishment of Membership Functions of Linguistic Terms**

In order to compute the MFs of the linguistic terms, the definition of TFNs as shown in Equation 1 was used. The MFs of each term in the Fuzzy Probability Scale are as follows:

$$\mu_{IPR}(x) = \begin{cases} \frac{x - 0.07}{0.15 - 0.07}, & \text{for } 0.07 \le x \le 0.15, \\ \frac{0.22 - x}{0.22 - 0.15}, & \text{for } 0.15 \le x \le 0.22, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mu_{IPS}(x) = \begin{cases} \frac{0.13 - x}{0.13}, & \text{for } 0 \le x \le 0.13, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mu_{UNC}(x) = \begin{cases} \frac{x - 0.19}{0.06}, & \text{for } 0.19 \le x \le 0.25, \\ \frac{0.37 - x}{0.12}, & \text{for } 0.25 \le x \le 0.37, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mu_{FCH}(x) = \begin{cases} \frac{x - 0.34}{0.16}, & \text{for } 0.34 \le x \le 0.5, \\ \frac{0.59 - x}{0.09}, & \text{for } 0.5 \le x \le 0.59, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mu_{EXP}(x) = \begin{cases} \frac{x - 0.58}{0.17}, & \text{for } 0.58 \le x \le 0.75, \\ \frac{0.78 - x}{0.03}, & \text{for } 0.75 \le x \le 0.78, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mu_{PRO}(x) = \begin{cases} \frac{x - 0.77}{0.08}, & \text{for } 0.77 \le x \le 0.85, \\ \frac{0.92 - x}{0.07}, & \text{for } 0.85 \le x \le 0.92, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mu_{CER}(x) = \begin{cases} \frac{x - 0.9}{0.1}, & \text{for } 0.9 \le x \le 1.0, \\ 0, & \text{otherwise.} \end{cases}$$

In general, the constructed TFN varies for each probability linguistic term. The TFN are verified based on a few characteristics listed below, which are the properties of fuzzy numbers, shape, overlap and vagueness. A consistency analysis of fuzzy MFs is described in the following section to further validate the TFNs.

**Consistency Analysis of Fuzzy Membership Functions**

Since MFs are a reflection of subjective human judgments, their properties and forms are often based on assumptions, with parameters estimated by domain experts. It is crucial to measure the consistency among these experts' MFs to ensure the reliability and robustness of the fuzzy model. A high degree of consistency validates that the model accurately captures a shared understanding or consensus among experts, making its outputs more credible and trustworthy. In this study, a pairwise consistency index measures the overlap between the MFs of two experts for a particular linguistic term $l$, represented as $k_{uv}^{l}$ was calculated as referred to in Equation 2 (Deng & Shi, 2003). The numerator calculates the integral of the minimum values of the two MFs, which represents their overlap. The denominator calculates the integral of the maximum values, representing the union of the two MFs. The overall consistency is taking the average of all pairs of experts.

$$k_{uv}^{l} = S\left(\tilde{A}_u, \tilde{A}_v\right)_l = \frac{1}{p}\sum S\left(\tilde{A}_u, \tilde{A}_v\right)_{u,v}^{l} \tag{11}$$

where $p$ is the number of unique expert pairs. The result yields a value between 0 and 1, where a value close to 1 indicates that the two fuzzy sets are highly similar (i.e., high consistency), while a value close to 0 indicates that the fuzzy sets are very different (i.e., low consistency). The result of the overall consistency index for each linguistic term is displayed in Table 5. The overall consistency indices for all linguistic terms are reasonably high, with most of them approaching 1. It indicates a high level of agreement among the experts, validating the reliability of the elicited MFs.

**Table 5**

*Consistency Analysis of Linguistic Terms*

| Linguistic Terms | Overall Consistency Index, $k_{uv}$ |
|---|---|
| Impossible (IPS) | 0.96995 |
| Improbable (IPR) | 0.722857 |
| Uncertain (UNC) | 0.801514 |
| Fair-chance (FCH) | 0.81409 |
| Expected (EXP) | 0.974757 |
| Probable (PRO) | 0.962248 |
| Certain (CER) | 0.986152 |

**Expert Feedback Validation**

While many studies rely on expert judgment to construct fuzzy MFs, little attention is given to their validation. This crucial process ensures that the MFs are not just mathematically sound but also meaningfully represent the context-dependent linguistic terms used by experts in a real-world emergency. The most reliable option, therefore, is to validate against human experts in the domain

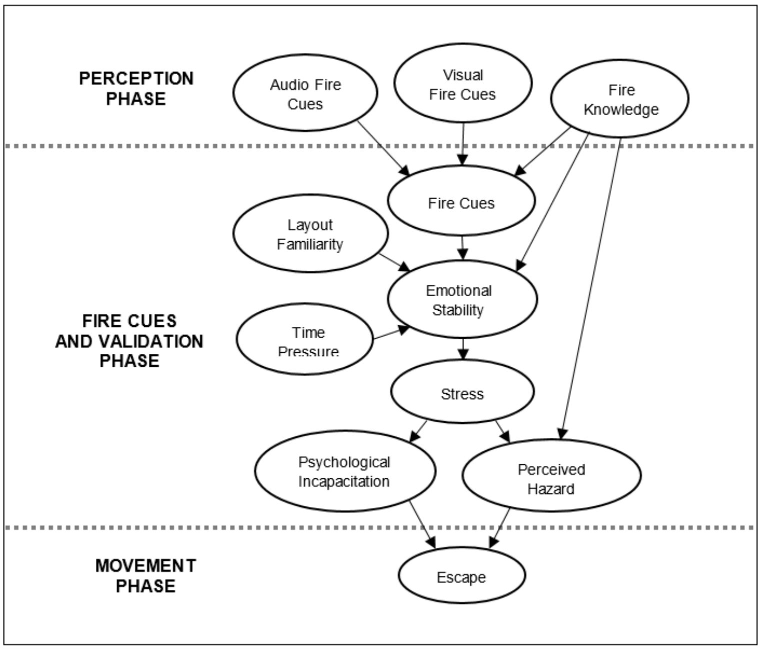
(Mosqueira-Rey et al., 2008). In this study, expert feedback was sought to validate the interpretability of the MFs of the Fuzzy Probability Scale within the EDM scenario. The expert validation revealed that the variation in the spread of terms in the fuzzy conversion scale reflects the inherent uncertainty and variability within this context. Terms with narrower spreads, such as "Probable" (0.77, 0.85, 0.92) and "Certain" (0.90, 1.0, 1.0), denote high confidence and decisiveness, representing stages where individuals have clear risk awareness and are ready to act. These narrower ranges capture consistent psychological certainty and are suitable for modelling situations with less ambiguity. Conversely, broader spreads like "Uncertain" (0.19, 0.25, 0.37) and "Fair-Chance" (0.34, 0.50, 0.59) reflect greater variability in decision-making, highlighting stages where individuals are influenced by ambiguous cues or external factors like social dynamics or environmental changes. These broader ranges are critical for representing the diversity of interpretations and responses in the face of unclear risk. This validation demonstrates that the scale's strength lies in its ability to encompass both the stable and uncertain phases of a response, rendering it a robust tool for modelling complex decision-making in emergency situations.

**Integration into Decision Models**

The Fuzzy Probability Scale is integrated into applicable EDM models to facilitate their practical application. In this study, the applicability of the developed scale is demonstrated using a BN decision model for emergency evacuation, specifically the PRiF (Psychological Response in a Fire) model. The model developed by Ramli et al. (2021) captures the causal relationships of psychological responses during the initial stages of fire events. By incorporating the elicited MFs into the PRiF model, the study aims to demonstrate how expert-derived linguistic terms, now quantified through fuzzy sets, can directly assist in the precise quantification of the model's probabilistic relationships. Figure 8 illustrates the PRiF model's structure and variables, with further details available in Ramli et al. (2021).

**Figure 8**

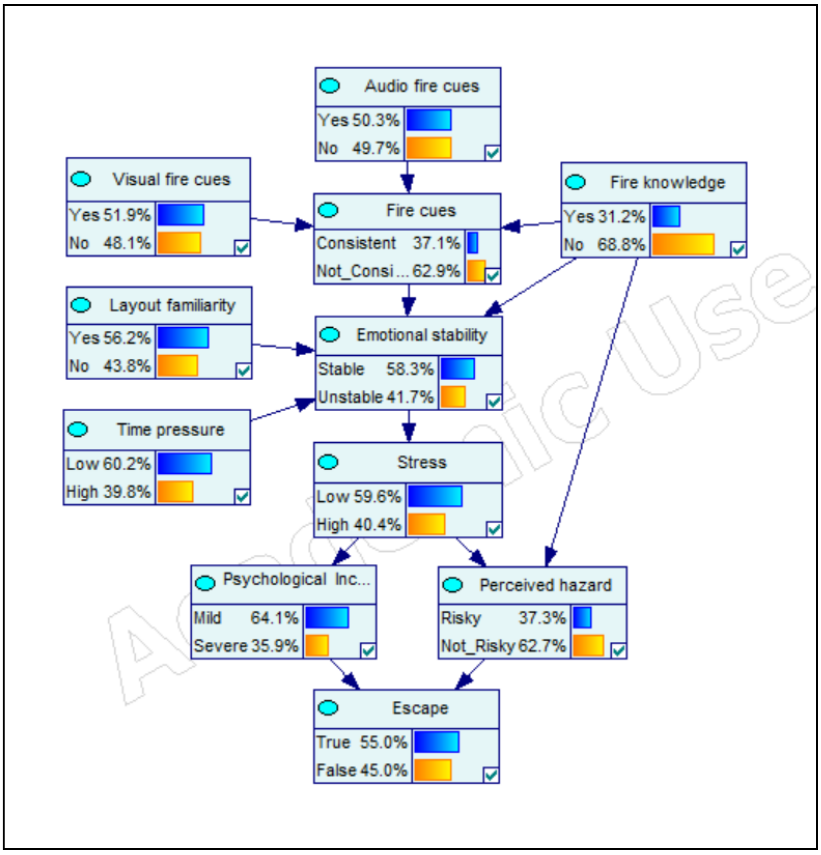
*Bayesian Network Model of PRiF from Ramli et al. (2021)*



Experts utilise the developed scale to infer their belief regarding the parameter values of the PRiF model. The experts select the most suitable linguistic terms, along with their corresponding MFs, to represent the likelihood of occurrence for variables in the model. The MFs of the linguistic terms are then converted into a crisp probability value, enabling the derivation of all model parameters. Consequently, these expert-provided probabilities enable a comprehensive understanding of evacuees' psychological responses during a fire event. Utilising the GeNIe Modeller (BayesFusion, n.d.), an open-source BN software, the marginal probabilities of variables in the PRiF model are presented graphically in Figure 9. These probabilities suggest that evacuees perceived the ability to perceive visual (51.9%), audio (50.3%) fire cues, even with no prior knowledge of fire (68.8%) eventually lead to a higher probability in perceiving inconsistencies of fire cues (62.9%), emotionally stable (58.3%), low stress level (59.6%), mild psychological incapacitation (64.1%) and perceiving hazard as not risky (62.7%). Finally, the evacuees have a higher probability of escape (55%) as compared to not escaping (45%) during a fire evacuation.

**Figure 9**

*Probabilities for Variables in the PRiF Model Generated in Genie Modeller*




Having obtained the above probabilities, the proposed fuzzy BN model can now be used to conduct various types of analysis using the inference mechanism. For example, let's suppose an individual failed to perceive fire cues, represented as a ‘Not consistent’ level in the ‘Fire Cues’ variable. To understand what might have caused this, the likelihood for the root nodes, ‘Audio Fire Cues’, ‘Visual Fire Cues’, and ‘Fire Knowledge’ being in their worst-case state of ‘No=2’ given the outcome. This needs to calculate the posterior probability $P(AC_2|FC_2)$, $P(VC_2|FC_2)$ and $P(FK_2|FC_2)$ using conventional Bayesian rules. For example, the posterior probability of $P(AC_2|FC_2)$ is calculated as follows:

$$P(AC_2|FC_2) = \frac{P(AC_2) \times P(AC_2, FC_2)}{P(FC_2)} = 0.59$$

Based on the analysis, the likelihood of not perceiving audio fire cues is 59%, given that inconsistent fire cues were observed. Similarly, the $P(VC_2|FC_2) = 0.55$ and $P(FK_2|FC_2) = 0.80$ are obtained. Comparing the posterior probabilities with their prior probabilities ($P(AC_2) = 0.497, P(VC_2) = 0.481$, and $P(FK_2) = 0.688$ (the initial probabilities from Figure 1), it can be seen that there is a significant change in the occurrence likelihood of 'Audio Fire Cues' (increased by 18.7%), 'Visual Fire Cues' (increased by 14.3%), and 'Fire Knowledge' (increased by 16.3%) in the event of an inconsistent perception of fire cues. It suggests that the 'Fire Cues' variable is highly sensitive to its parent nodes. These results suggest that to ensure successful fire cue perception within an emergency information system, it is essential to enhance the delivery of audio and visual fire cues and ensure that users possess sufficient fire knowledge. This might involve a well-designed dashboard with prominent visual and audio alerts or providing mandatory pre-emergency information and training to users. To further justify these conclusions, a sensitivity analysis will be conducted in the next section.

## Sensitivity Analysis

Sensitivity analysis for fuzzy BN measures how a model's performance is affected by minor changes to its input parameters. This analysis is crucial for investigating the effects of inaccuracies or incompleteness in the model's parameters on its output. The most common approach to performing this analysis is to alter parameter values and then monitor the effects on posterior probabilities using an evidence propagation method (Pitchforth & Mengersen, 2013; Ren et al., 2008, 2009). In this case study, the preliminary conclusion that the 'Fire Cues' node is sensitive to its root nodes 'Audio Fire Cues' (AC), 'Visual Fire Cues' (VC), and 'Fire Knowledge' (FK) is drawn based on posterior probabilities ($P(AC_2|FC_2)$, $P(VC_2|FC_2)$ and $P(FK_2|FC_2)$. Therefore, a key aspect of the sensitivity analysis is to analyse how these probabilities change when prior probabilities take on different values.

Without loss of generality, each of the TFNs $P_f(AC_2)$, $P_f(VC_2)$, and $P_f(FK_2)$ take five different values, ranging from (0.30, 0.40, 0.50) to (0.75, 0.80, 0.85) (see Table 6). These fuzzy numbers were then defuzzified using the Center of Area (CoA) method to obtain crisp values. These crisp values were then used in a BN analysis to calculate the posterior probabilities. To save computation time, these results can be obtained using Genie software.

**Table 6**

*Sensitivity Analysis Results between Prior and Posterior Probabilities*

| No | Fuzzy Prior Probabilities<br>$P_f(AC_2)$ | Crisp Prior Probabilities and Posterior Probabilities<br>$P(AC_2)$ | <br>$P(AC_2\|FC_2)$ | Change Between Prior and Posterior Probabilities (%) |
|---|---|---|---|---|
| 1 | (0.30, 0.40, 0.50) | 0.4 | 0.49 | 22.50% |
| 2 | (0.35, 0.45, 0.65) | 0.48 | 0.57 | 18.75% |
| 3 | (0.30, 0.55, 0.70) | 0.52 | 0.61 | 17.31% |
| 4 | (0.45, 0.60, 0.75) | 0.6 | 0.68 | 13.33% |
| 5 | (0.40, 0.65, 0.80) | 0.62 | 0.7 | 12.90% |
| | | | Average change (%) = | 16.96% |

(continued)

| | Fuzzy Prior Probabilities | Crisp Prior Probabilities and Posterior Probabilities | | Change Between Prior and Posterior Probabilities (%) |
|---|---|---|---|---|
| No | $P_f(VC_2)$ | $P(VC_2)$ | $P(VC_2 \mid FC_2)$ | |
| 1 | (0.33, 0.41, 0.55) | 0.43 | 0.500 | 16.28% |
| 2 | (0.30, 0.5, 0.60) | 0.47 | 0.540 | 14.89% |
| 3 | (0.35, 0.50, 0.65) | 0.5 | 0.570 | 14.00% |
| 4 | (0.38, 0.53, 0.71) | 0.54 | 0.610 | 12.96% |
| 5 | (0.42, 0.60, 0.75) | 0.58 | 0.650 | 12.07% |
| | | | Average change (%) = | 14.04% |
| No | $P_f(FK_2)$ | $P(FK_2)$ | $P(FK_2 \mid FC_2)$ | |
| 1 | (0.40, 0.60, 0.80) | 0.60 | 0.74 | 23.33% |
| 2 | (0.45, 0.65, 0.85) | 0.65 | 0.78 | 20.00% |
| 3 | (0.48, 0.78, 0.85) | 0.70 | 0.81 | 15.17% |
| 4 | (0.50, 0.75, 0.90) | 0.72 | 0.83 | 15.81% |
| 5 | (0.75, 0.80, 0.85) | 0.80 | 0.88 | 10.00% |
| | | | Average change (%) | 16.86% |

The results were shown in Table 6. As can be seen in the last column of Table 6, the change between prior and posterior probabilities clearly indicates that there is a significant change between $P_f(AC_2)$ and $P(AC_2|FC_2)$ (average change is 16.96%), and also between $P_f(FK_2)$ and $P(FK_2|FC_2)$ (average change is 16.86%). Meanwhile, the change between $P_f(VC_2)$ and $P(VC_2|FC_2)$ (average change is 14.04%) is less significant than those of AC and FK. Therefore, there is reason to believe that the conclusions made earlier are reliable and the assigned probabilities in BN are rational. In essence, variations in expert-provided parameters do not significantly alter the outcome of the posterior probability. It also indicate that the PRiF model's parameters are stable, robust, and the assigned probabilities are rational. This confirms the rationality and meaningfulness of the quantitative information provided by experts during the elicitation process, specifically through the use of the developed Fuzzy Probability Scale.

## CONCLUSION

This study presented a comprehensive framework for the expert elicitation of fuzzy linguistic MFs from domain experts using a graphic survey based on the IE method. The primary contribution lies in the systematic extraction of expert knowledge and its quantification into meaningful MFs, enabling the straightforward application of TFNs to define linguistic labels for emergency decision-making. The integration of these MFs into a BN decision model, specifically the PRiF model, and subsequent performance evaluation, demonstrated the practical application and effectiveness of the framework in a complex EDM scenario. Furthermore, the developed scale's interpretability, emphasised through expert feedback validation, makes it uniquely aligned with the decision-maker's terminology and reasoning. These results suggest that this framework, with its emphasis on expert-driven fuzzy linguistic quantification, can be effectively employed in EDM where expert knowledge is crucial for handling uncertainty and complexity. This study presents a significant advancement in emergency management decision systems, offering a robust methodology for incorporating expert uncertainty into fuzzy-based DSS. Future research can extend this framework by incorporating real-world datasets to enhance validation and test its versatility in other EDM contexts. The methodology could also be enhanced to support expert systems, creating more dynamic decision-making tools that continuously update based on real-time data or expert input. These directions would further solidify the framework's role in advancing decision support systems for various high-stakes fields.

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