



How to cite this article:

Yab, L. Y., Wahid, N., & Hamid, R. A. (2024). Impact of balanced exploration and exploitation on high-dimensional feature selection with hierarchical whale optimisation algorithm. *Journal of Information and Communication Technology*, 23(4), 593-626. <https://doi.org/10.32890/jict2024.23.4.2>

Impact of Balanced Exploration and Exploitation on High-dimensional Feature Selection with Hierarchical Whale Optimisation Algorithm

¹Li Yu Yab, ²Noorhaniza Wahid & ³Rahayu A Hamid
Faculty of Computer Science and Information Technology,
Universiti Tun Hussein Onn Malaysia, Malaysia

¹hi220017@student.uthm.edu.my

*²nhaniza@uthm.edu.my

³rahayu@uthm.edu.my

*Corresponding author

Received: 3/9/2024 Revised: 15/10/2024 Accepted: 15/10/2024 Published: 28/10/2024

ABSTRACT

High-dimensional datasets (HDDs) pose significant challenges in feature selection due to their complex nature. While metaheuristic algorithms like the Whale Optimisation Algorithm (WOA) have shown promise in addressing these challenges, stability issues are least addressed. Through an extensive literature review, it was identified that stability is manifested in the equilibrium of exploration and exploitation. Hence, this study introduced the Hierarchical Whale Optimisation Algorithm (HiWOA), an approach designed to enhance the WOA's stability and performance in HDD feature selection tasks.

The HiWOA incorporates a two-phase strategy comprising a nonlinear control parameter based on the arcsine function and a hierarchical position-update mechanism adapted from the Grey Wolf Optimiser. The proposed HiWOA was evaluated through 23 benchmark optimisation functions and feature selection experiments on 11 medical HDDs. The results indicate that the HiWOA outperformed the WOA and a modified variant (mWOA) in terms of a better fitness value, a more balanced exploration-exploitation ratio, and improved classification accuracy with fewer selected features. These findings demonstrate the HiWOA's effectiveness in enhancing stability, making it a robust solution for high-dimensional optimisation and feature selection.

Keywords: Balancing exploration and exploitation, control parameter, feature selection, stability, whale optimisation algorithm.

INTRODUCTION

In recent years, high-dimensional datasets (HDDs), defined as datasets with a high number of features rather than the number of observations (Bolón-Canedo et al., 2016), have become more common in data mining (Debata & Mohapatra, 2021; Zhao et al., 2024). The noise and irrelevant features in HDDs complicates classification tasks (Hasan et al., 2023). As the number of features increases, computational demands also rise, emphasising the need for advanced techniques to overcome the curse of dimensionality (Bolón-Canedo et al., 2016). Feature selection is a key approach for dimensionality reduction in HDDs (X. Li et al., 2024), offering various methods, such as filter-based, wrapper-based, and embedded approaches (Theng & Bhojar, 2023). Remarkably, metaheuristic algorithms, such as Particle Swarm Optimisation (PSO), the Grey Wolf Optimiser (GWO), and the Whale Optimisation Algorithm (WOA), have gained prominence for their superior performance in feature selection and classification tasks (Basir et al., 2019; Kazerani, 2024; Kumar & Kumar, 2021; Y. Wang et al., 2024; Yab et al., 2023). These algorithms help efficiently navigate feature spaces and select high-quality subsets, improving both the accuracy and efficiency of data mining.

Among metaheuristic techniques, the WOA is notable for mimicking the foraging behaviour of whales (Mirjalili & Lewis, 2016). However, like other metaheuristics, the WOA faces challenges, such as slow convergence, getting stuck in local optima, and an imbalance between

exploration and exploitation (Nadimi-Shahraki et al., 2023). Though modifications have been proposed to improve the WOA in feature selection (Elmogly et al., 2023; F. Wang et al., 2023; Yab et al., 2022, 2024), the issues of its stability have received less attention. The stability of an algorithm in feature selection indicates sensitivity to changes in input data during the process of identifying important features (Khaire & Dhanalakshmi, 2022). However, the underlying algorithmic factors contributing to stability issues remain unclear. In addition to that, stability issues in metaheuristic algorithms, particularly the WOA, are a crucial yet often overlooked factor in feature selection for HDDs. As HDDs become more prevalent in data mining, these stability challenges grow more significant. Therefore, while the WOA excels at emulating whales' foraging behaviour, neglecting algorithmic stability issues could impact its overall performance.

Recognising the importance of the WOA's stability, especially in feature selection for HDDs, this study aimed to improve the WOA's stability in feature selection for HDDs by proposing a two-phase hierarchical strategy called the Hierarchical Whale Optimisation Algorithm (HiWOA). The HiWOA incorporates a nonlinear control strategy and adopts the social hierarchy concept from the GWO, focusing on improving the search strategy and striking a better balance between exploration and exploitation.

This paper is organised as follows. In the Introduction section, the research problem and objectives are outlined. The next section reviews related works on metaheuristic algorithms, the WOA, and feature selection for HDDs. Following this, the background concepts of WOA, GWO, and the design of proposed HiWOA are detailed in the Proposed Hierarchical Whale Optimisation Algorithm section. In the subsequent section, the Evaluation and Results describes the experimental setup and discusses the results by comparing the performances of the HiWOA with those of the WOA and mWOA. Finally, the study concluded with a summary of the findings.

RELATED WORKS

Stability from Various Schools of Thought

The concept of stability in metaheuristic algorithms can be somewhat unclear and loosely defined as it relates to various underlying

challenges, such as convergence, local optima avoidance, and the balance between exploration and exploitation. In response, this study aimed to provide a more nuanced understanding of stability by drawing on insights from multiple perspectives in the literature. In terms of convergence, one study linked stability to the convergence speed of an algorithm, suggesting that a stable algorithm converges more quickly to a solution (Reddy & Saha, 2022). On the contrary, another study suggested an algorithm is said to be stable when it converges at a slower rate (Shehab et al., 2018). Other researchers added that stability is reflected in the consistency of convergence curves, where a stable algorithm shows a steady performance (Wu et al., 2019).

Apart from convergence, stability is also linked with local optima avoidance. For instance, a research study emphasised that stability involves an algorithm's ability to avoid local optima and prevent stagnation in suboptimal solutions (Han et al., 2023). In addition to that, a study suggested that stability is achieved when an algorithm can avoid local optima by swiftly returning to the vicinity of the optimal solution after experiencing fluctuations during iterations (F. Wang et al., 2023). Despite varying perspectives, the predominant view in the literature is that stability is fundamentally about achieving a balance between exploration and exploitation (Cheng et al., 2023; Mohammed & Rashid, 2020; Tiwari & Chaturvedi, 2022; Z. Wang et al., 2023). Exploration allows the algorithm to search widely for new solutions, while exploitation refines known solutions. Too much exploration can hinder convergence, while too much exploitation may result in an algorithm getting stuck in local optima. Stability is thus defined by an algorithm's ability to navigate this tradeoff effectively, ensuring a reliable performance.

To sum up, this study redefined stability in the WOA as an improved balance between exploration and exploitation, enhanced convergence speed, and a robust capability to avoid local optima. From the synthesis of various perspectives from the literature, it has been highlighted that a stable algorithm should not focus too much on exploration or exploitation but rather on the balance between them. Therefore, a quantitative measure of the algorithm's stability was introduced based on the tradeoff between exploration and exploitation.

Stability Issue Workarounds

Several works in the literature addressed the above-mentioned underlying issues of stability. For instance, stability has been

improved by adjusting the parameters in an algorithm. A research study proposed a variant of the WOA with opposition-based learning called opposition-based WOA (OWOA) to enhance convergence and balance exploration and exploitation (F. Wang et al., 2023). By using the sine function to adjust the spiral coefficient b , the OWOA showed improved performance on 12 benchmark functions and power flow optimisation problems. Despite outperforming the original WOA and other algorithms, the OWOA showed unstable performance metrics, indicating a need for better stability and exploration of multi-objective applications. To address the WOA's slow convergence in HDD feature selection, another research study proposed a modified WOA (mWOA) by linearly increasing the control parameter, a , from 0 to 2 (Yab et al., 2022). The results showed improved accuracy over the standard WOA, though further convergence speed and accuracy enhancements are needed. A derived work from the mWOA reported that the algorithm worked slightly better with wrapper-based than filter-based feature selection in terms of accuracy (Yab et al., 2024). However, the stability of the algorithm was not investigated. In addition, an adaptive nonlinear WOA (ANWOA) was proposed to address slow convergence and local optima stagnation (Elmogly et al., 2023). It uses two control parameters with exponential and dynamic ranges and a dynamic inertia weight for position updates. Tested on 23 benchmark functions and CEC2020, the ANWOA showed improved accuracy, stability, and convergence speed, with future works suggested to explore parameter effects and real-world applications.

Researchers have explored similar themes in studies related to the WOA and across various other metaheuristic algorithms. A study introduced a modified version of the Tunicate Swarm Algorithm, incorporating a nonlinear convergence factor to improve the exploration-exploitation balance (Liu et al., 2023). The nonlinear convergence factor effectively enhanced the balance in the Seagull Optimisation Algorithm (Y. Li et al., 2023). Likewise, Wu et al. (2019) introduced a nonlinear control strategy based on the arcsine function (NCS-arcsin) to improve the balance between exploration and exploitation. They demonstrated that the standard linear control strategy failed to capture the nonlinear nature of whale predation and that the arcsine function outperformed cosine, sine, exponential, and logarithmic functions in optimisation. Therefore, the present study adopted the nonlinear control strategy, specifically using the arcsine function as the primary approach to improve the WOA's stability.

Other than adjusting the control parameters in an algorithm, the hybridisation or adaptation approach is also a common theme in the literature for solving stability-related problems. It is noteworthy that researchers often pair algorithms with complementary strengths to enhance the overall performance, demonstrating the effectiveness of such strategies in addressing stability issues. The Crow Search Algorithm (CSA) was combined with the Bald Eagle Search Algorithm (BESA) to improve the BESA's exploitation efficiency by leveraging the CSA's exploration strengths (Nirmal et al., 2024). An improved WOA by adopting the Equilibrium Optimiser (EO), called the EWOA, was proposed to address slow convergence and balance exploration and exploitation (Tan & Mohamad-Saleh, 2023). The EWOA combines the WOA's encircling and bubble-net attacking mechanisms with the EO's weight balance strategy. Tested on CEC2017, CEC2019, and CEC2023 benchmark functions, the EWOA demonstrated robustness and faster convergence. Another enhanced WOA, also called the EWOA, was proposed to address slow convergence due to poor exploitation (Reddy & Saha, 2022). By modifying position update equations, inspired by the Artificial Bee Colony algorithm, and increasing exploration randomness, this version of the EWOA achieved the best results in 7 of 10 benchmark functions and effectively solved pressure vessel optimisation problems. However, the high number of whales in the algorithm led to increased computational complexity. Therefore, the WOA might need another algorithm with strength in exploitation. A promising candidate is the GWO, which, when hybridised with the Fireworks Algorithm (FA), showed improvements over the original FA due to the GWO's strong exploitation capabilities (Yue et al., 2020). Table 1 shows the summary of the findings of the literature.

In summary, the literature emphasises that stability challenges involve balancing exploration and exploitation, improving convergence, and avoiding local optima. Various strategies have been suggested, such as adjusting control parameters and using complementary algorithms. This study aimed to enhance the WOA's stability by combining these approaches.

Table 1

Summary of Literature Analysis

Authors	Issues to solve	Proposed method	Research outcome
Wu et al. (2019)	<ul style="list-style-type: none"> - Linear control strategy failing to capture nature of whale predation - Slow convergence - Fall into local optima 	Nonlinear control strategy based on arcsine function NCS-arcsin	Better balance between exploration and exploitation
Yue et al. (2020)	<ul style="list-style-type: none"> - Slow convergence - Fall into local optima 	GWO + Fireworks Algorithm	Improved over original FA due to GWO's strong exploitation capabilities
Reddy and Saha (2022)	<ul style="list-style-type: none"> - Slow convergence - Weak exploitation capability - Weak stability 	Enhanced WOA + Artificial Bee Colony	Improved convergence and stability but high computational complexity
Yab et al. (2022)	<ul style="list-style-type: none"> - Slow convergence - High computational time in HDDs 	Modified WOA (mWOA) with linear control strategy from 0 to 2	Higher convergence speed and accuracy
Elmogy et al. (2023)	<ul style="list-style-type: none"> - Fall into local optima - Poor search update techniques 	Adaptive nonlinear WOA (ANWOA) using two control parameters with exponential and dynamic inertia weight for position updates	Improved convergence, stability, and local optima avoidance
F. Wang et al. (2023)	<ul style="list-style-type: none"> - Slow convergence - Fall into local optima 	WOA with opposition-based learning (OWOA) uses sine function to adjust spiral coefficient,	Higher convergence speed and accuracy

(continued)

Authors	Issues to solve	Proposed method	Research outcome
Liu et al. (2023)	<ul style="list-style-type: none"> - Premature convergence - Low accuracy 	Modified Tunicate Swarm Algorithm with nonlinear control strategy	Better balance between exploration and exploitation
Tan and Mohamad-Saleh (2023)	<ul style="list-style-type: none"> - Less efficient optimisation - High computational complexity 	WOA + Equilibrium Optimiser (EO)	Better optimisation efficiency and balance between exploration and exploitation
Y. Li et al. (2023)	<ul style="list-style-type: none"> - Slow convergence - Fall into local optima - Low accuracy 	Modified Seagull Optimisation Algorithm with nonlinear control strategy	Improved stability
Nirmal et al. (2024)	<ul style="list-style-type: none"> - Weak exploration capability 	Crow Search Algorithm (CSA) + Bald Eagle Search Algorithm (BESA)	Improved over original BESA due to CSA's strong exploration capabilities

THE PROPOSED HIERARCHICAL WHALE OPTIMISATION ALGORITHM

This section describes the proposed HiWOA, which enhances the standard WOA with a two-phase hierarchical strategy. It details the introduction of a nonlinear control parameter and a hierarchical position-update mechanism inspired by the GWO.

Basic Concept of Whale Optimisation Algorithm

The WOA is a metaheuristic technique inspired by the hunting strategies of humpback whales (Mirjalili & Lewis, 2016). The WOA falls within the swarm intelligence category of metaheuristic taxonomy (Rajwar et al., 2023). The whales swim in a spiral pattern while blowing bubbles to trap prey. This behaviour is mathematically modelled for optimisation, utilising a population of search agents (whales) to iteratively find the best solution. The three core phases of the WOA—exploration, exploitation, and encirclement—are explored in the next subsections.

Exploration

In the exploration phase, the whales move toward random positions in the search space, adding diversity to the population and mimicking their random search for prey. This phase occurs when $|\vec{A}|$ is greater than or equal to 1, as defined in Equation 1. Vector \vec{A} is a coefficient vector determined by the linearly decreasing parameter \vec{a} from Equation 2 and a random number of \vec{r} . The whale's position is updated using the position of a randomly selected whale, as shown in Equation 3 and Equation 4, where \vec{X} represents the whale's position, \vec{X}_{rand} is the position of the randomly chosen whale, and \vec{D} is the distance between them.

$$\vec{A} = 2\vec{a} \cdot \vec{r} - \vec{a} \quad (1)$$

$$a = 2 - t(2/MaxIter) \quad (2)$$

$$\vec{D} = |\vec{C} \cdot \vec{X}_{rand} - \vec{X}| \quad (3)$$

$$\vec{X}(t + 1) = \vec{X}_{rand} - \vec{A} \cdot \vec{D} \quad (4)$$

Exploitation

In the exploitation phase, the whales move closer to the best-known solution using two strategies: the shrinking mechanism and the spiral route. In the shrinking mechanism, the whales adjust their positions between their current spot and the best solution, refining their approach over time. The spiral route, defined in Equation 6, models a logarithmic spiral, where \vec{X}^* represents the best-known position and \vec{D}^i is the distance to it, as shown in Equation 5. The whales alternate between these two strategies with a 50 percent probability, as expressed in Equation 7, depending on a random value of p .

$$\vec{D}^i = |\vec{X}^*(t) - \vec{X}(t)| \tag{5}$$

$$\vec{X}(t + 1) = \vec{D}^i \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t) \tag{6}$$

$$\vec{X}(t + 1) = \begin{cases} \vec{X}^*(t) - \vec{A} \cdot \vec{D} & \text{if } p < 0.5 \\ \vec{D}^i \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t) & \text{if } p \geq 0.5 \end{cases} \tag{7}$$

Encirclement

The encirclement phase refines the search by guiding the whales closer to the best solution, ensuring convergence. All search agents adjust their positions toward the best agent identified. This is mathematically expressed in Equation 9 and Equation 10, where \vec{C} is the coefficient vector from Equation 8, t is the current iteration, \vec{X} is the whale's current position, and \vec{X}^* represents the best-known position. Distance \vec{D} indicates the distance between the whale and the optimal position. If a better solution is found in any iteration, \vec{X}^* will be updated.

$$\vec{C} = 2 \cdot \vec{r} \tag{8}$$

$$\vec{D} = |\vec{C} \cdot \vec{X}^*(t) - \vec{X}(t)| \tag{9}$$

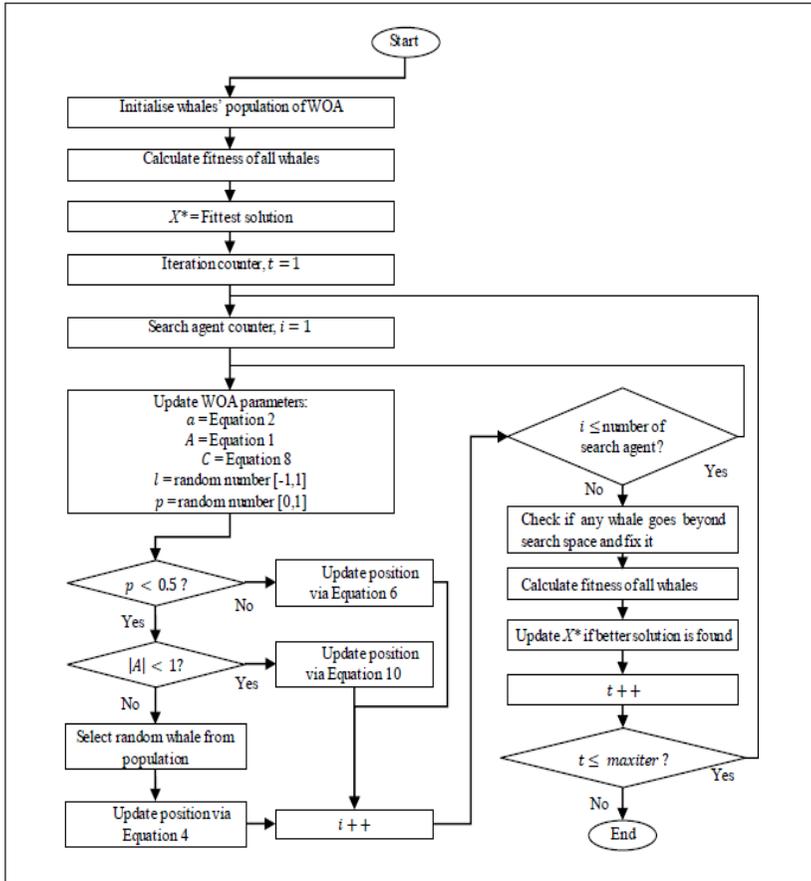
$$\vec{X}(t + 1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D} \tag{10}$$

In summary, the WOA's effectiveness hinges on its ability to balance exploration and exploitation, driven by its position update mechanism

and control parameter. These features allow the WOA to explore the search space and converge toward optimal solutions. Figure 1 outlines the overall working structure of the WOA, providing a visual representation of the algorithm's workflow.

Figure 1

WOA's Workflow



Basic Concept of Grey Wolf Optimiser

The Grey Wolf Optimiser (GWO) is an established metaheuristic algorithm developed to tackle optimisation problems due to its unique hunting strategy (Mirjalili et al., 2014). The GWO mimics the social hierarchy of grey wolves during group hunting, consisting of four

ranks: alpha (α), beta (β), delta (δ), and omega (ω). The alpha wolf leads, with the beta wolf following the alpha's lead and dominating the delta and omega wolves. The delta wolf, in turn, is dominated by the alpha and beta wolves but holds authority over the lowest-ranked omega wolf, which follows the top three wolves.

In the GWO, the best solution is represented by the alpha wolf, the second-best by the beta wolf, and the third-best by the delta wolf, with all other solutions labelled as omega wolves. Unlike the WOA, which relies on a single leader, the GWO enhances its search process by incorporating the top three solutions, which are the alpha, beta, and delta wolves, into its exploration strategy. Although the GWO and WOA share similar parameters of C , A , and a , their primary difference lies in their position update mechanisms. The GWO updates positions during the hunting process using Equation 11–13. The overall workflow of the GWO is presented in Figure 2.

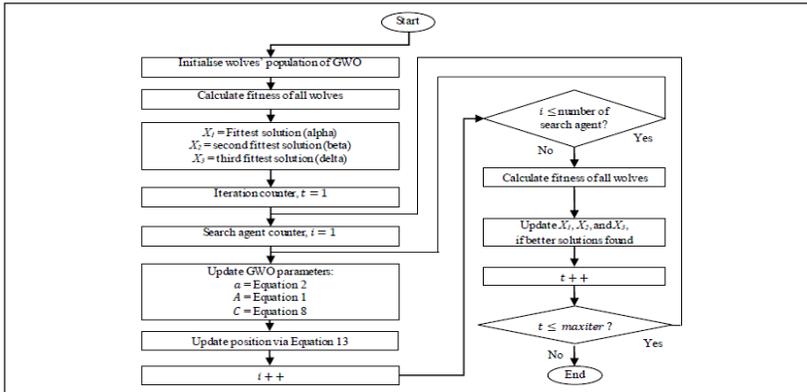
$$\begin{aligned} \vec{D}_\alpha &= |\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}|, \vec{D}_\beta = |\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}|, \vec{D}_\delta = |\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}| \end{aligned} \tag{11}$$

$$\begin{aligned} \vec{X}_1 &= \vec{X}_\alpha - \vec{A}_1 \cdot \vec{D}_\alpha, \vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot \vec{D}_\beta, \vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot \vec{D}_\delta \end{aligned} \tag{12}$$

$$\vec{X}(t + 1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{13}$$

Figure 2

GWO's Workflow

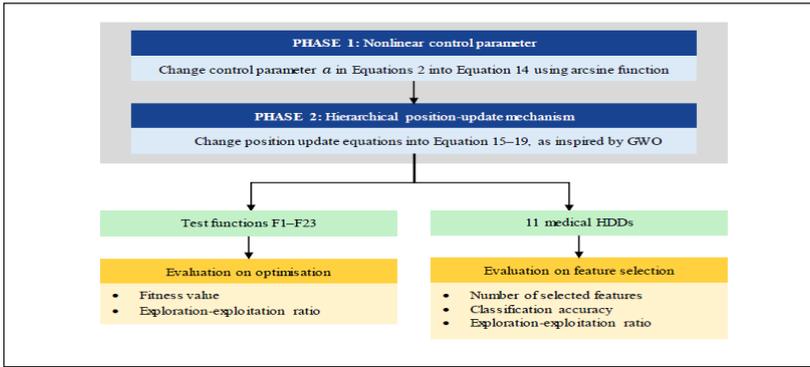


The Design of HiWOA

The HiWOA adopts the GWO’s social hierarchy, where wolves of different ranks lead the pack. As a result, the term “hierarchical” in the HiWOA reflects this concept. Figure 3 depicts the framework of this study.

Figure 3

The Research Framework of the Proposed HiWOA



Phase 1: Nonlinear Control Parameter

To address the overall stability issue of the WOA regarding the balance between exploration and exploitation phases, the control parameter, a , needs improvement. It was originally defined in Equation 2 as a linear control strategy, where it linearly decreases from 2 to 0 over the iterations (Mirjalili & Lewis, 2016). However, a linear control strategy may not accurately represent the complex dynamics of exploration and exploitation, as whales’ natural behaviour does not follow a strictly linear pattern. Relying solely on a linear control strategy is, therefore, insufficient. Hence, this study introduced a nonlinear control strategy using the arcsine function inspired by Wu et al. (2019) via Equation 14.

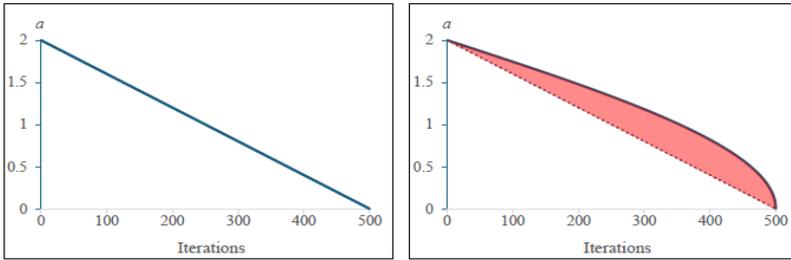
$$a = 2 - \frac{2 \arcsine\left(\frac{t}{\maxiter}\right)}{\arcsine(1)} \quad (14)$$

The HiWOA introduces an arcsine function to modify the rate of decrease, resulting in a more curved, upward-sloping gradient. The initial and final values remain fixed at 2 and 0, respectively (Mirjalili

& Lewis, 2016). The formula is derived as shown in Equation 14 to ensure the correct progression of the control parameter's decline. Figure 4 shows a comparison of the control parameter strategies of the original WOA and the proposed HiWOA, where the number of iterations is set as 500 (Mirjalili & Lewis, 2016).

Figure 4

Comparison of Different Control Strategies



(a) Linear control strategy in WOA

(b) Nonlinear control strategy in HiWOA

The nonlinear strategy in the HiWOA prioritises a steady growth of exploration in early iterations and a steeper turn to exploitation in later iterations. This modification aims to achieve a more rational tradeoff between exploration and exploitation to symbolise the complexity of the search-hunt balance of actual whales. It seeks to improve convergence speed, avoid local optima, and enhance the overall stability of the HiWOA. The nonlinear control strategy in the HiWOA, as shown in Figure 4(b), increases the control parameter's value more gradually in early iterations compared with the linear strategy in the WOA. This gradual increase allows for extended exploration, which helps the algorithm search a wider area of the solution space before converging. As a result, the HiWOA would be less likely to get trapped in local optima, hence improving the algorithm's ability to find the global optimum. In later iterations, the nonlinear control strategy shifts more steeply towards exploitation as the control parameter's value decreases more rapidly. This sharper decline enhances the algorithm's convergence speed by focusing the search on the best solutions found during the exploration phase. By doing so, the HiWOA can effectively refine these solutions, leading to a more accurate and faster convergence.

Overall, the use of the arcsine function in the control parameter ensures that the HiWOA maintains a balanced approach between exploration

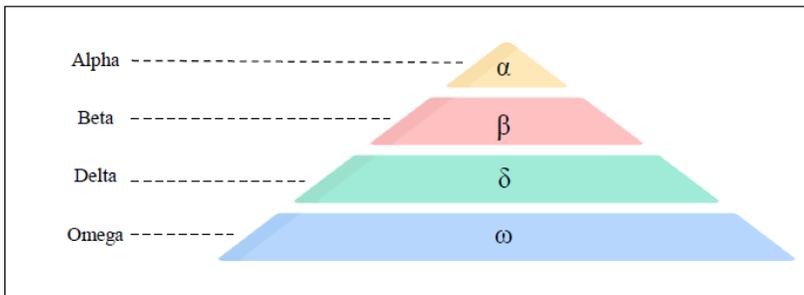
and exploitation. This balance is crucial for avoiding premature convergence to suboptimal solutions and ensures the algorithm's stability across different optimisation and feature selection problems. The arced transition between the exploration and exploitation phases aims for a more robust and reliable search process, ultimately enhancing the overall stability of the HiWOA.

Phase 2: Hierarchical Position-Update Mechanism

In this phase, the proposed HiWOA with the arcsine nonlinear control strategy from Phase 1 is further modified with a hierarchical position-update mechanism. This is achieved by adapting the concept of social hierarchy from the GWO to enhance exploitation capabilities. The use of three leaders aims to enhance the HiWOA's effectiveness compared with that of the WOA with its single leader by promoting a better balance between exploration and exploitation. The GWO's social hierarchy, which involves alpha, beta, and delta leaders directing the rest, offers a more robust exploitation approach than the WOA's single-leader approach. Figure 5 illustrates the social hierarchy of Grey Wolf.

Figure 5

Grey Wolf's Social Hierarchy



Having multiple leaders allows the proposed HiWOA to pull search agents in various directions, reducing the risk of premature convergence to local optima. This hierarchical guidance enables the algorithm to explore a broader search space, while maintaining a balanced decision-making process, as agents are not solely dependent on one dominant leader, which could lead to biased results. Additionally, the presence of multiple leaders can help refine the search more effectively, as it

minimises the chance of overlooking promising regions of the search space, hence resulting in improved exploitation without getting stuck early in the optimisation process.

In short, mimicking the hierarchical social structure promotes a more balanced exploration and exploitation. Therefore, by integrating the GWO's hierarchy, the proposed HiWOA can obtain the best solutions given by the top three whales. To implement the hierarchical structure of the search agents in the HiWOA, the position update mechanism is updated accordingly. Equation 5 is updated into Equation 15 to compute the distances between the current whale and the top three fittest whales rather than just the best solution. Similarly, Equation 6 and Equation 7 are replaced by Equation 16 and Equation 17, respectively.

$$\vec{D}_\alpha = |\vec{X}_\alpha - \vec{X}_t|; \vec{D}_\beta = |\vec{X}_\beta - \vec{X}_t|; \vec{D}_\delta = |\vec{X}_\delta - \vec{X}_t| \tag{15}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}$$

where,

$$\begin{aligned} \vec{X}_1 &= \vec{D}_\alpha \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}_\alpha(t) \\ \vec{X}_2 &= \vec{D}_\beta \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}_\beta(t) \\ \vec{X}_3 &= \vec{D}_\delta \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}_\delta(t) \end{aligned} \tag{16}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}$$

where,

$$\begin{aligned} \vec{X}_1 &= \begin{cases} \vec{X}_\alpha(t) - \vec{A} \cdot \vec{D}_\alpha & \text{if } p < 0.5 \\ \vec{D}_\alpha \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}_\alpha(t) & \text{if } p \geq 0.5 \end{cases} \\ \vec{X}_2 &= \begin{cases} \vec{X}_\beta(t) - \vec{A} \cdot \vec{D}_\beta & \text{if } p < 0.5 \\ \vec{D}_\beta \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}_\beta(t) & \text{if } p \geq 0.5 \end{cases} \\ \vec{X}_3 &= \begin{cases} \vec{X}_\delta(t) - \vec{A} \cdot \vec{D}_\delta & \text{if } p < 0.5 \\ \vec{D}_\delta \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}_\delta(t) & \text{if } p \geq 0.5 \end{cases} \end{aligned} \tag{17}$$

Besides modifying exploitation, the encirclement mechanism is also changed by updating Equation 9 into Equation 18 to calculate the distances between the current search agent and the top three fittest solutions. Furthermore, Equation 10 is replaced by Equation 19 to

reflect hierarchical encirclement based on the average distance to these top three solutions.

$$\overrightarrow{D}_\alpha = |\overrightarrow{C}_1 \cdot \overrightarrow{X}_\alpha - \overrightarrow{X}_t|; \overrightarrow{D}_\beta = |\overrightarrow{C}_2 \cdot \overrightarrow{X}_\beta - \overrightarrow{X}_t|; \overrightarrow{D}_\delta = |\overrightarrow{C}_3 \cdot \overrightarrow{X}_\delta - \overrightarrow{X}_t| \quad (18)$$

$$\overrightarrow{X}(t+1) = \frac{\overrightarrow{X}_1 + \overrightarrow{X}_2 + \overrightarrow{X}_3}{3}$$

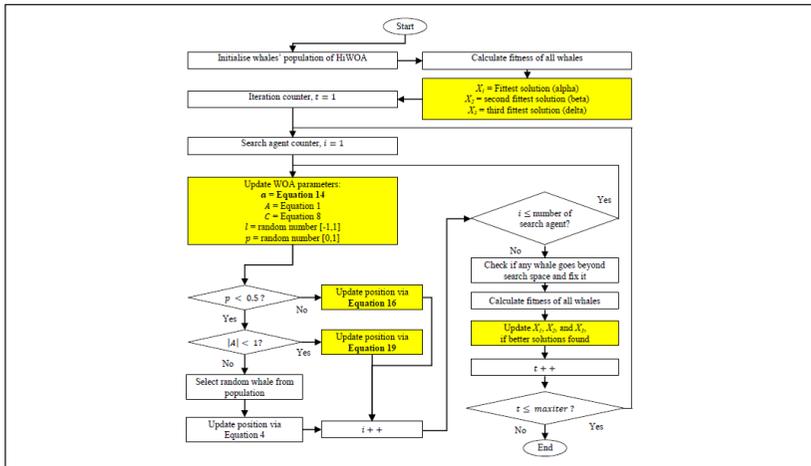
where, (19)

$$\begin{aligned} \overrightarrow{X}_1 &= \overrightarrow{X}_\alpha(t) - \overrightarrow{A} \cdot \overrightarrow{D}_\alpha \\ \overrightarrow{X}_2 &= \overrightarrow{X}_\beta(t) - \overrightarrow{A} \cdot \overrightarrow{D}_\beta \\ \overrightarrow{X}_3 &= \overrightarrow{X}_\delta(t) - \overrightarrow{A} \cdot \overrightarrow{D}_\delta \end{aligned}$$

In general, the proposed HiWOA maintains the spiral-shaped route of the WOA but incorporates a hierarchical position-update mechanism to improve exploitation and encirclement. This enhancement aims to strengthen local search capabilities to help strike a balance between exploration and exploitation, hence improving the algorithm's stability. Figure 6 shows the proposed HiWOA's workflow. The performance of the proposed HiWOA was tested in optimisation and then further validated in feature selection. The next section presents the evaluation and results.

Figure 6

HiWOA's Workflow



EVALUATION AND RESULTS

This section outlines the experimental setup used to evaluate the performance of the proposed HiWOA. The experiments were designed to assess the algorithm's effectiveness in both optimisation tasks and feature selection problems. The setup details, comprising the functions, datasets, and evaluation metrics, are presented to ensure a comprehensive analysis of the algorithm's capabilities.

Experimental Setup for Optimisation

In this subsection, the optimisation tasks used to benchmark the HiWOA against the WOA and mWOA are discussed, detailing the test functions and parameter settings to evaluate the algorithm's performance in optimisation problems. The optimisation experiments used 23 benchmark functions (Mirjalili & Lewis, 2016) divided into three categories: unimodal (F1–F7) for testing convergence to a single global optimum, multimodal (F8–F13) for evaluating navigation through multiple local optima, and fixed-dimension multimodal (F14–F23) for assessing performance in complex, higher-dimensional spaces. This study ran experiments on all test functions individually for the WOA, the mWOA, and the proposed HiWOA. All experiments were performed using MATLAB R2023b on a system with an Intel Core i7-10750H CPU, 32 GB of RAM, and an NVIDIA GeForce RTX 3060 GPU. Each experiment was run 30 times with up to 500 iterations and 30 solutions per iteration. The same setup was applied across both optimisation and feature selection to ensure consistency in the evaluation.

The evaluation criteria were the fitness value, exploration percentage ($Xpl\%$), and exploitation percentage ($Xpt\%$). The stability of a search algorithm depends on the balance between exploration and exploitation, where the ideal ratio of the exploration and exploitation phases should be closer to 50:50 (Tzanetos & Dounias, 2021). This study used the dimension-wise diversity measurement from Hussain et al. (2019) to quantify this balance. Equation 20 calculates the average diversity across dimensions, where $median(x^j)$ is the median of dimension j across the swarm, x_i^j is the j -th dimension of the i -th individual, and n is the swarm size. The algorithm's exploration and exploitation percentages can be determined using Equation 21 by tracking swarm diversity per iteration. Here, Div represents diversity

in each iteration, Div_{max} is the maximum observed diversity, and $Xpl\%$ and $Xpt\%$ indicate the exploration and exploitation percentages for each iteration, respectively. These metrics allowed for a detailed analysis of the algorithm’s efficiency, convergence behaviour, and balance between the exploration and exploitation phases.

$$Div = \frac{1}{D} \sum_{j=1}^D Div_j, \text{ where } Div_j = \tag{20}$$

$$\frac{1}{n} \sum_{i=1}^n median(x^j) - x_i^j \tag{21}$$

$$Xpl\% = \frac{Div}{Div_{max}} \times 100 ; Xpt\% = \frac{|Div - Div_{max}|}{Div_{max}} \times 100$$

100

Experimental Setup for Feature Selection

In this section, the focus is on evaluating the performance of the proposed HiWOA in selecting relevant features from HDDs. This study utilised 11 medical-related HDDs obtained from Blake & Merz (1998) and Zhu et al. (2007). The HDD description is presented in Table 2. The term “number of instances” in Table 2 indicates the number of rows in a dataset, while “number of features” represents the number of columns. “Dimensionality” is the product of these rows and columns. The “number of classes” denotes the type of classification, where “2” stands for binary classification, and a number greater than 2 indicates multi-class classification. The datasets in Table 2 are sorted in descending order by dimensionality.

Table 2

HDD Description

Dataset	No. of instances	No. of features	No. of classes	Dimensionality
RNA-Seq	801	20531	5	16445331
Ovarian	253	15154	2	3833962
SMK_CAN_187	187	19993	2	3738691
Breast	97	24481	2	2374657
Lung	181	12533	2	2268473
GLI_85	85	22283	2	1894055
MLL	72	12582	3	905904
CNS	60	7129	2	427740
Lymphoma	62	4026	3	249612
SRBCT	83	2308	4	191564
colon	62	2000	2	124000

The feature selection process utilised the k -Nearest Neighbour (k NN) classifier with $k = 5$. The datasets were split into 80 percent for training and 20 percent for testing. The dimension for each dataset corresponds to the number of features present in the dataset. Each experiment was conducted with 30 individual runs, with up to 500 iterations per run and 30 whales as search agents per iteration. The evaluation criteria for feature selection were the number of features selected, classification accuracy, and the balance between exploration and exploitation ($Xpl\%:Xpt\%$) from Equation 21. Accuracy was calculated using Equation 22, where TP and TN represent positive and negative instances correctly classified, while FP and FN denote incorrectly classified instances.

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN} \quad (22)$$

Optimisation Performance

In this subsection, the results of the optimisation experiments are discussed, including the performance of the proposed HiWOA against those of the WOA and mWOA on various benchmark functions. The optimisation results are presented in Table 3. The fitness values presented in Table 3 revealed that the proposed HiWOA algorithm outperformed both the WOA and mWOA across 9 test functions. In comparison, the WOA ranked first in 8 functions, while the mWOA led in 7. Additionally, there is one case where the HiWOA surpassed the WOA, even if it did not achieve the top ranking. Although the differences in performance may appear marginal, these results suggest that the HiWOA demonstrated superior overall performance compared with those of both the WOA and mWOA. The average fitness value results further underscored the HiWOA's effectiveness. With an average fitness value of $6.5100E+02$, the HiWOA significantly outperformed both the WOA ($2.0102E+03$) and mWOA ($1.5791E+03$). This notable difference indicated that the HiWOA was more efficient in finding optimal solutions by demonstrating superior convergence capabilities. The lower fitness value suggested that the HiWOA effectively minimised the objective function across various test functions, improving overall performance.

Regarding the $Xpl\%:Xpt\%$ results in Table 3, the WOA and mWOA led in 7 and 6 test functions, respectively. On the other hand, the

proposed HiWOA demonstrated notable strength by achieving a more balanced ratio, outperforming both the WOA and mWOA in 10 test functions. When comparing the HiWOA directly with its original form, the WOA, the proposed HiWOA excelled in 14 test functions, indicating its superiority in maintaining a balanced exploration-exploitation tradeoff among the three algorithms. In addition, the average exploration-exploitation ratios for all test functions revealed a clear winner among the three algorithms. The HiWOA's ratio of $Xpl\%:Xpt\%$ (31.26: 68.74) represented a slight yet significant balanced ratio compared with those of the WOA (30.44: 69.56) and mWOA (30.75: 69.25). Although the HiWOA's $Xpl\%:Xpt\%$ ratio was not exactly 50:50, its closer alignment to a more balanced ratio suggested it was more adept at maintaining stability in its search process. A balanced ratio promotes stability by ensuring that an algorithm explores a broad range of potential solutions while also intensifying its search in promising areas. This stability is crucial for avoiding premature convergence and achieving a more thorough search, ultimately leading to more robust and optimal solutions. Further analysis of the HiWOA's performance is presented in the next subsection, where it was applied to feature selection problems.

Table 3*Comparison of the Optimisation Results*

Function	Fitness value						
	WOA	mWOA	HiWOA	WOA	mWOA	HiWOA	
F1	6.2211E-48	7.3537E-50*	8.1207E-20	26.38:73.62	25.58:74.42	26.50:73.50*	
F2	1.7864E-37	3.1224E-38*	2.8906E-18	23.95:76.05*	21.82:78.18	23.60:76.40	
F3	5.6062E+04	4.4935E+04	2.6288E+04*	39.22:60.78*	39.11:60.89	36.35:63.65	
F4	5.3948E+01	6.4877E+01	8.0461E+00*	37.65:62.35	38.52:61.48*	33.40:66.60	
F5	2.8135E+01	2.7928E+01*	2.8208E+01	25.16:74.84	29.74:70.26*	27.95:72.05#	
F6	5.1624E-01*	1.1017E+00	8.5277E-01	28.93:71.07*	27.79:72.21	26.19:73.81	
F7	3.7099E-03*	6.4514E-03	3.8612E-03	32.99:67.01	34.01:65.99*	33.75:66.25#	
F8	-9.8872E+03	-8.6900E+03	-1.1324E+04*	32.45:67.55	35.47:64.53*	30.97:69.03	
F9	5.6843E-15*	2.0843E-14	3.0316E-14	29.66:70.34	29.75:70.25	30.46:69.54*	
F10	4.5889E-15	3.7600E-15*	1.5362E-11	27.30:72.70	26.95:73.05	28.16:71.84*	
F11	0.0000E+00*	0.0000E+00*	1.1664E-02	28.21:71.79	32.11: 67.89*	30.68:69.32#	
F12	6.0339E-02	3.5802E-02*	7.9290E-02	29.87:70.13*	29.21:70.79	27.31:72.69	
F13	5.8259E-01*	1.0410E+00	7.9480E-01	26.44:73.56	30.29:69.71*	28.94:71.06#	
F14	3.3841E+00	4.8175E+00	1.8474E+00*	33.46:66.54*	31.60:68.40	32.40:67.60	
F15	1.0775E-03	6.9225E-04	5.2378E-04*	29.04:70.96	29.11:70.89	30.65:69.35*	
F16	-1.0316E+00*	-1.0316E+00	-1.0316E+00	32.97:67.03*	32.97:67.03	32.64:67.36	
F17	3.9790E-01*	3.9791E-01	4.0291E-01	31.38:68.62	32.89:67.11	33.66:66.34*	

(continued)

Function	Fitness value					
	WOA	mWOA	HiWOA	WOA	mWOA	HiWOA
F18	3.0001E+00*	3.0003E+00	3.2296E+00	33.31:66.69*	32.10:67.90	31.08:68.92
F19	-3.8238E+00	-3.8575E+00*	-3.8566E+00#	30.47:69.53	30.83:69.17	36.28:63.72*
F20	-3.1971E+00	-3.2163E+00	-3.2179E+00*	31.53:68.47	31.18:68.82	35.76:64.24*
F21	-7.9385E+00	-6.1738E+00	-9.1673E+00*	30.12:69.88	29.37:70.63	32.91:67.09*
F22	-7.0188E+00	-6.7404E+00	-8.3952E+00*	30.36:69.64	29.31:70.69	33.75:66.25*
F23	-6.7208E+00	-8.2014E+00	-8.8866E+00*	29.18:70.82	27.47:72.53	35.68:64.32*
Total best	8	7	9 (10)	7	6	10 (14)
Average	2.0102E+03	1.5791E+03	6.5100E+02	30.44:69.56	30.75:69.25	31.26:68.74

Note: * best result; # HiWOA outperformed WOA; F1-F7 = unimodal; F8-F13 = multimodal; F14-F23 = fixed-dimension multimodal

Feature Selection Performance

This subsection presents the findings from the feature selection experiments, focusing on how well the proposed algorithm performed in selecting relevant features from the HDDs. The results are presented in Table 4. Based on the results in Table 4, which show the number of selected features, the WOA and mWOA achieved top rankings in 3 datasets. On the other hand, the proposed HiWOA outperformed both, securing the top position in 5 datasets, making it the most effective algorithm for dimensionality reduction by selecting the fewest yet most relevant features. Furthermore, the HiWOA performed better in 8 out of 11 datasets when compared with the WOA. The results demonstrate the HiWOA's superiority in handling HDDs by consistently selecting the fewest features, especially in the top few rows of Table 4, where the datasets had the highest dimension, such as RNA-Seq, Ovarian, SMK_CAN_187, Breast, etc. This makes the HiWOA particularly adept at feature selection tasks in HDDs. Besides that, the average number of selected features showed that the HiWOA excels in dimensionality reduction, as the HiWOA selected an average of 137 features, compared with 232 for the WOA and 213 for the mWOA. By reducing the number of features needed, the HiWOA improves efficiency and performance, making the model simpler and more focused on important information.

Moreover, the classification accuracy percentages shown in Table 4 clearly established the HiWOA's dominance, as it consistently achieved the highest accuracy across all datasets. In contrast, the WOA and mWOA only managed to secure the top spot in 3 datasets and 1 dataset, respectively. The HiWOA's effectiveness was further highlighted by its outstanding performance in datasets with the highest dimensions, consistently delivering the highest accuracy. Also, the results reveal that the HiWOA achieved an average accuracy of 97.12 percent across all HDDs, which was higher than the WOA's 95.93 percent and the mWOA's 95.46 percent. It shows that the HiWOA excels in selecting a minimal number of features while achieving high classification accuracy. The features chosen by the HiWOA were compact, highly relevant, and of exceptional quality, making the HiWOA the most effective algorithm for classification tasks in HDDs.

Furthermore, the results for the ratio of $X_{pl}\%: X_{pt}\%$ demonstrate that the HiWOA significantly outperformed both the WOA and mWOA

across all datasets. The average ratios revealed a marked improvement with the HiWOA, which had a ratio of 30.25 percent exploration to 69.75 percent exploitation. In contrast, the WOA's average ratio was 9.44 percent exploration to 90.56 percent exploitation, and the mWOA's average ratio was 18.50 percent exploration to 81.50 percent exploitation. This substantial difference indicated that the HiWOA enhanced the balance between exploration and exploitation and achieved this balance more effectively than the other algorithms. The higher exploration percentage in the HiWOA suggested that it better explored the search space while maintaining strong exploitation capabilities. The improved balance enhances the algorithm's overall stability and performance, making the HiWOA a more robust and efficient solution for high-dimensional optimisation and feature selection tasks.

Table 4

Comparison of Feature Selection Results

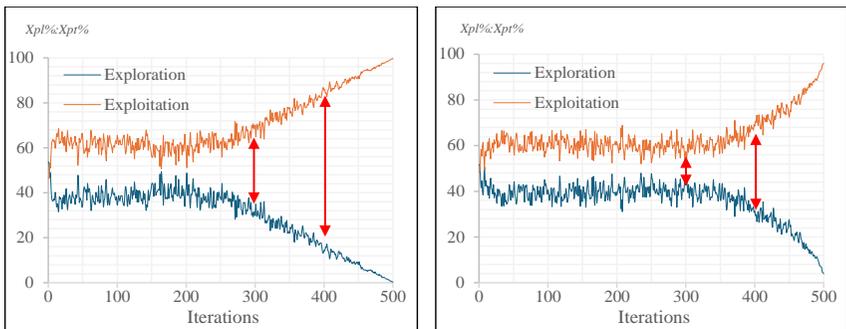
Dataset	Number of features selected			Classification accuracy (%)					
	WOA	mWOA	HiWOA	WOA	mWOA	HiWOA			
RNA-Seq	98	111	83*	100*	99.98	100*	7.39:92.61	15.60:84.40	29.73:70.27*
Ovarian	23	31	10*	99.07	98.67	99.73*	6.01:93.99	19.33:80.67	27.59:72.41*
SMK_CAN_187	445	110*	195#	84.23	84.68	86.67*	9.99:90.01	17.32:82.68	30.15:69.85*
Breast	726	110*	179#	86.84	84.39	90.88*	10.33:89.67	20.13:79.87	34.91:65.09*
Lung	652*	836	701	97.83	98.25	99.17*	14.23:85.77	19.79:80.21	26.21:73.79*
GLI_85	106	392	94*	98.82	98.24	99.41*	8.88:91.12	20.17:79.83	30.36:69.64*
MLL	77	178	47*	100*	100*	100*	10.64:89.36	20.02:79.98	27.93:72.07*
CNS	355	507	131*	91.94	93.33	93.89*	10.70:89.30	20.29:79.71	33.25:66.75*
Lymphoma	3*	4	4	98.46	98.46	99.74*	4.13:95.87	13.38:86.62	34.13:65.87*
SRBCT	49*	56	57	100*	99.58	100*	9.88:90.12	20.22:79.78	28.35:71.65*
colon	15	8*	9#	98.06	94.44	98.89*	11.72:88.28	17.26:82.74	30.10:69.90*
Total best	3	3	5 (8)	3	1	11	0	0	11
Average	232	213	137	95.93	95.46	97.13	9.44: 90.56	18.50: 81.50	30.25: 69.75

Note: * best result; # HiWOA outperformed WOA

In summary of the evaluation results from Table 3 and Table 4, the HiWOA excels in both optimisation and feature selection. Although there were cases where the HiWOA's improvements over the WOA seemed marginal and sometimes on par (e.g., the HiWOA achieving the same accuracy of 100 percent as the WOA in RNA-Seq, MLL, and SRBCT datasets), the HiWOA was still considered better, as it selected fewer features and achieved a more balanced $Xpl\%:Xpt\%$. With the average exploration-exploitation ratios of 31.26:68.74 in optimisation and 30.25:69.75 in feature selection, the HiWOA significantly outperformed the WOA and mWOA, which had lower exploration rates. This improved balance was due to the HiWOA's hierarchical structure and nonlinear control strategy based on the arcsine function. The hierarchical mechanism enhanced exploitation, while the nonlinear control strategy refined the exploration-exploitation tradeoff throughout the iterations. Figure 7 and Figure 8 illustrate the HiWOA's better balance compared with the WOA in both optimisation and feature selection tasks.

Figure 7

Comparison of Exploration-Exploitation Ratios ($Xpl\%:Xpt\%$) in) Optimisation (Function F23)

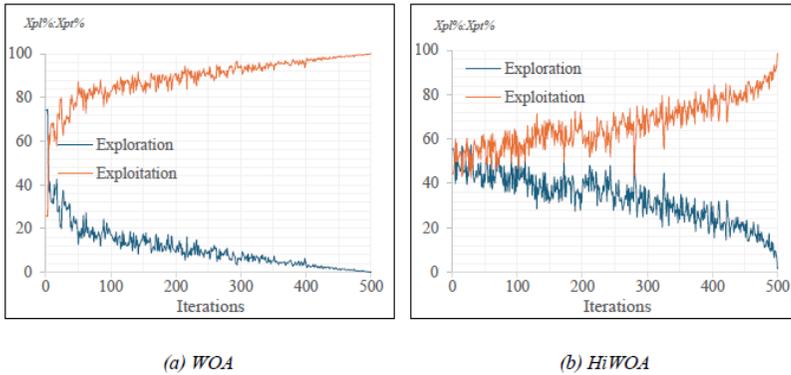


(a) WOA

(b) HiWOA

Figure 8

Comparison of Exploration-Exploitation Ratios ($Xpl\%:Xpt\%$) in Feature Selection (CNS Dataset)



For the WOA (Figure 7(a) and Figure 8(a)), the exploration-exploitation ratios showed a noticeable gap, with exploration dominating the early stages and exploitation being less pronounced. This imbalance suggested that the WOA tends to focus more on exploring the search space, which can lead to less efficient exploitation of promising solutions. As a result, while the algorithm might avoid local optima, it may struggle with refining solutions and converging quickly to the global optimum. This uneven distribution between exploration and exploitation phases can hinder the algorithm's overall optimisation performance, particularly in optimisation and feature selection tasks.

In contrast, the HiWOA demonstrated much closer ratios between exploration and exploitation, as shown in Figures 7(b) and 8(b). This indicated a more balanced search process, where both phases worked in tandem. The more even distribution allowed the HiWOA to explore new areas of the search space, while also refining and exploiting promising solutions efficiently. The improved balance enhanced the algorithm's overall performance, reducing the likelihood of getting stuck in local optima and improving convergence towards the global optimum. Unlike the 50:50 ratio suggested by Tzanetos and Dounias (2021) as an optimal tradeoff for algorithm performance, this study proved that the HiWOA, with an average $Xpl\%:Xpt\%$ ratio closer to 30:70, significantly outperformed both the WOA and mWOA. This finding challenges the conventional view by showing that a greater emphasis on exploitation does not necessarily hinder exploration but

enhances overall algorithm effectiveness. The HiWOA's ability to maintain a more stable and balanced search process, as evidenced by its exploration and exploitation ratios, allowed it to refine solutions found, leading to superior performance in optimisation and feature selection tasks.

CONCLUSION

This study introduces the HiWOA as an enhancement to the standard WOA for addressing stability issues in HDD feature selection by offering five key contributions. The first contribution provides a clearer interpretation of stability, linking it to the balance between exploration and exploitation, as well as the algorithm's convergence and its ability to avoid local optima. Second, the HiWOA is introduced to improve the WOA's stability, aiming to tackle optimisation and feature selection tasks better. Third, an arcsine function is used in the HiWOA to modify control parameter a nonlinearly, which allows for a better tradeoff between exploration and exploitation. Fourth, the incorporation of the social hierarchy concept from the GWO introduces three leader-whales, improving solution quality by replacing the single-leader approach. Finally, the evaluation results proved that the HiWOA outperformed the WOA and mWOA in both optimisation and feature selection.

The reason for enhancing the WOA by introducing the HiWOA is to address the WOA's struggle with balancing exploration and exploitation, which slows convergence and risks getting stuck in local optima. These issues reduce its stability, especially in high-dimensional feature selection tasks. The HiWOA addresses these problems by introducing a nonlinear control strategy and a hierarchical position-update mechanism to improve search efficiency. The proposed HiWOA was tested on 23 optimisation functions and 11 HDDs for k NN wrapper feature selection against the WOA and mWOA. The evaluation results show that the HiWOA outperformed both the WOA and mWOA across various optimisation functions and feature selection scenarios in terms of the fitness value, number of selected features, classification accuracy, and exploration-exploitation

ratio of $X_{pl}\%:X_{pt}\%$ using the same experiment settings. It was found that the introduction of a nonlinear control parameter using the arcsine function, together with the adaptation of a hierarchical position-update mechanism, contributed to the HiWOA's improved performance. The HiWOA showed evidence of maintaining a better balance between exploration and exploitation, with an average $X_{pl}\%:X_{pt}\%$ ratio of 30.25:69.75 for HDD feature selection tasks, while the WOA and mWOA only achieved 9.44:90.56 and 18.50:81.50, respectively. With a balance closer to 30:70, the HiWOA achieved better stability and higher-quality results, confirming that the proposed method effectively addresses the identified issues.

Nonetheless, the HiWOA has a limitation, where its evaluation results are marginal compared with the WOA's. Thus, the tradeoff between exploration and exploitation can be further improved. For future works, it is suggested that the control parameter a in the HiWOA undergoes more testing by using trigonometry functions other than the arcsine and investigating the influence on the algorithm's performance. Further simulations with different settings are worth exploring to better understand the broader applicability and robustness of the proposed HiWOA.

ACKNOWLEDGMENT

This research was supported by the Ministry of Higher Education (MOHE) through the Fundamental Research Grant Scheme (FRGS/1/2023/ICT02/UTHM/02/6).

REFERENCES

- Basir, M. A., Yusof, Y., & Saifullah, M. (2019). Basir, M. A., Yusof, Y., & Hussin, M. S. (2019). Optimisation of attribute selection model using bio-inspired algorithms. *Journal of Information and Communication Technology, 18*(1), 35-55. <https://doi.org/10.32890/jict2019.18.1.3>
- Blake, C. L., & Merz, C. J. (1998). *UCI Machine Learning Repository*. <https://archive.ics.uci.edu/ml/index.php>

- Bolón-Canedo, V., Sánchez-Maróño, N., & Alonso-Betanzos, A. (2016). Feature selection for high-dimensional data. *Progress in Artificial Intelligence*, 5, 65-75. <https://doi.org/10.1007/978-3-319-21858-8>
- Cheng, S., Zhang, M. M., Shi, Y. H., Lu, H., Lei, X. J., & Wang, R. (2023). Generalised pigeon-inspired optimisation algorithm for balancing exploration and exploitation. *Zhongguo Kexue Jishu Kexue/Scientia Sinica Technologica*, 53(2), 268–279. <https://doi.org/10.1360/SST-2021-0371>
- Debata, P. P., & Mohapatra, P. (2021). Selection of informative genes from high-dimensional cancerous data employing an improvised meta-heuristic algorithm. *Evolutionary Intelligence*, 0123456789. <https://doi.org/10.1007/s12065-021-00593-y>
- Elmogy, A., Mqrish, H., Elawady, W., & El-Ghaish, H. (2023). ANWOA: An adaptive nonlinear whale optimisation algorithm for high-dimensional optimisation problems. *Neural Computing and Applications*, 35(30), 22671–22686. <https://doi.org/10.1007/s00521-023-08917-y>
- Han, C., Ranjun, W., Cheng, T., Liu, Z., & Wei, S. (2023). The stability analysis of particle swarm optimisation-grey wolf optimisation algorithm. In A. Bhattacharjya & X. Feng (Eds.), *International Conference on Computer, Artificial Intelligence, and Control Engineering (CAICE 2023)* (p. 164). SPIE. <https://doi.org/10.1117/12.2681160>
- Hasan, S. N. S., Jamil, N. W., & Ahmat, H. (2023). A review of dimension reduction techniques for classification on high-dimensional data. *AIP Conference Proceedings*, 2896(1). <https://doi.org/10.1063/5.0177327/2922033>
- Hussain, K., Salleh, M. N. M., Cheng, S., & Shi, Y. (2019). On the exploration and exploitation in popular swarm-based metaheuristic algorithms. *Neural Computing and Applications*, 31(11), 7665–7683. <https://doi.org/10.1007/s00521-018-3592-0>
- Kazerani, R. (2024). Improving breast cancer diagnosis accuracy by Particle Swarm Optimisation feature selection. *International Journal of Computational Intelligence Systems*, 17(1), 44. <https://doi.org/10.1007/s44196-024-00428-5>
- Khaire, U. M., & Dhanalakshmi, R. (2022). Stability investigation of improved Whale Optimisation Algorithm in the process of feature selection. *IETE Technical Review (Institution of Electronics and Telecommunication Engineers, India)*, 39(2), 286–300. <https://doi.org/10.1080/02564602.2020.1843554>

- Kumar, N., & Kumar, D. (2021). An improved grey wolf optimisation-based learning of artificial neural network for medical data classification. *Journal of Information and Communication Technology*, 20(2), 213-248. <https://doi.org/10.32890/jict2021.20.2.4>
- Li, X., Wang, J., Liu, Y., Song, H., Wang, Y., Hou, J.-N., Zhang, M., & Hao, W.-K. (2024). Classification feature selection and dimensionality reduction based on logical binary sine-cosine function arithmetic optimisation algorithm. *Egyptian Informatics Journal*, 26, 100472. <https://doi.org/10.1016/j.eij.2024.100472>
- Li, Y., Li, W., Yuan, Q., Shi, H., & Han, M. (2023). Multi-strategy improved Seagull Optimisation algorithm. *International Journal of Computational Intelligence Systems*, 16(1). <https://doi.org/10.1007/s44196-023-00336-0>
- Liu, G., Guo, Z., Liu, W., Cao, B., Chai, S., & Wang, C. (2023). MSHHOTSA: A variant of tunicate swarm algorithm combining multi-strategy mechanism and hybrid Harris optimisation. *PLoS ONE*, 18(8 August). <https://doi.org/10.1371/journal.pone.0290117>
- Mirjalili, S., & Lewis, A. (2016). The Whale Optimisation Algorithm. *Advances in Engineering Software*, 95, 51–67. <https://doi.org/10.1016/j.advengsoft.2016.01.008>
- Mirjalili, S., Mirjalili, S. M., & Lewis, A. (2014). Grey Wolf Optimiser. *Advances in Engineering Software*, 69, 46–61. <https://doi.org/10.1016/j.advengsoft.2013.12.007>
- Mohammed, H., & Rashid, T. (2020). A novel hybrid GWO with WOA for global numerical optimisation and solving pressure vessel design. *Neural Computing and Applications*, 32(18), 14701–14718. <https://doi.org/10.1007/s00521-020-04823-9>
- Nadimi-Shahraki, M. H., Zamani, H., Asghari Varzaneh, Z., & Mirjalili, S. (2023). A systematic review of the Whale Optimisation algorithm: Theoretical foundation, improvements, and hybridisations. *Archives of Computational Methods in Engineering*, 30(7), 4113–4159. <https://doi.org/10.1007/s11831-023-09928-7>
- Nirmal, A., Jayaswal, D., & Kachare, P. H. (2024). A Hybrid Bald Eagle-Crow Search Algorithm for Gaussian mixture model optimisation in the speaker verification framework. *Decision Analytics Journal*, 10. <https://doi.org/10.1016/j.dajour.2023.100385>

- Rajwar, K., Deep, K., & Das, S. (2023). An exhaustive review of the metaheuristic algorithms for search and optimisation: Taxonomy, applications, and open challenges. *Artificial Intelligence Review*, 56(11), 13187–13257. <https://doi.org/10.1007/s10462-023-10470-y>
- Reddy, K., & Saha, A. K. (2022). A modified Whale Optimisation algorithm for exploitation capability and stability enhancement. *Heliyon*, 8(10), e11027. <https://doi.org/10.1016/j.heliyon.2022.e11027>
- Shehab, M., Khader, A. T., & Laouchedi, M. (2018). A hybrid method based on Cuckoo Search algorithm for global optimisation problems. *Journal of Information and Communication Technology*, 17(3), 469-491. <https://doi.org/10.32890/jict2018.17.3.8261>
- Tan, W. H., & Mohamad-Saleh, J. (2023). A hybrid Whale Optimisation algorithm based on equilibrium concept. *Alexandria Engineering Journal*, 68, 763–786. <https://doi.org/10.1016/j.aej.2022.12.019>
- Theng, D., & Bhoyar, K. K. (2023). Feature selection techniques for machine learning: A survey of more than two decades of research. *Knowledge and Information Systems*, 66(3), 1575-1637. <https://doi.org/10.1007/s10115-023-02010-5>
- Tiwari, A., & Chaturvedi, A. (2022). A hybrid feature selection approach based on information theory and dynamic butterfly optimisation algorithm for data classification. *Expert Systems with Applications*, 196. <https://doi.org/10.1016/j.eswa.2022.116621>
- Tzanetos, A., & Dounias, G. (2021). Exploration and exploitation analysis for the sonar inspired optimisation algorithm. *Annals of Mathematics and Artificial Intelligence*, 89(8–9), 857–874. <https://doi.org/10.1007/s10472-021-09755-1>
- Wang, F., Feng, S., Pan, Y., Zhang, H., Bi, S., & Zhang, J. (2023). Dynamic spiral updating whale optimisation algorithm for solving optimal power flow problem. *Journal of Supercomputing*, 79(17), 19959–20000. <https://doi.org/10.1007/s11227-023-05427-5>
- Wang, Y., Ran, S., & Wang, G.-G. (2024). Role-oriented binary grey wolf optimiser using foraging-following and Lévy flight for feature selection. *Applied Mathematical Modelling*, 126, 310–326. <https://doi.org/10.1016/j.apm.2023.08.043>
- Wang, Z., Zhou, Y., Takagi, T., Song, J., Tian, Y. S., & Shibuya, T. (2023). Genetic algorithm-based feature selection with

- manifold learning for cancer classification using microarray data. *BMC Bioinformatics*, 24(1). <https://doi.org/10.1186/s12859-023-05267-3>
- Wu, X., Zhang, S., Xiao, W., & Yin, Y. (2019). The exploration/exploitation tradeoff in Whale Optimisation algorithm. *IEEE Access*, 7, 125919–125928. <https://doi.org/10.1109/ACCESS.2019.2938857>
- Yab, L. Y., Wahid, N., & Hamid, R. A. (2024). Improved ozone level detection through feature selection with modified Whale Optimisation algorithm. *Qubahan Academic Journal*, 4(1), 265–276. <https://doi.org/10.48161/qaj.v4n1a466>
- Yab, L. Y., Wahid, N., & Hamid, R. A. (2022). A modified Whale Optimisation algorithm as filter-based feature selection for high dimensional datasets. In *Lecture Notes in Networks and Systems: Vol. 457 LNNS*. https://doi.org/10.1007/978-3-031-00828-3_9
- Yab, L. Y., Wahid, N., & Hamid, R. A. (2023). Inversed control parameter in Whale Optimisation algorithm and Grey Wolf Optimiser for wrapper-based feature selection: A comparative study. *International Journal on Informatics Visualisation*, 7(2). <https://doi.org/10.30630/joiv.7.2.1509>
- Yue, Z., Zhang, S., & Xiao, W. (2020). A novel hybrid algorithm based on Grey Wolf Optimiser and Fireworks algorithm. *Sensors*, 20(7), Article 7. <https://doi.org/10.3390/s20072147>
- Zhao, J., Lv, S., Xiao, R., Ma, H., & Pan, J.-S. (2024). Hierarchical learning multi-objective firefly algorithm for high-dimensional feature selection. *Applied Soft Computing*, 165, 112042. <https://doi.org/10.1016/j.asoc.2024.112042>
- Zhu, Z., Ong, Y. S., & Dash, M. (2007). Markov blanket-embedded genetic algorithm for gene selection. *Pattern Recognition*, 40(11), 3236–3248. <https://doi.org/10.1016/J.PATCOG.2007.02.007>