

## **PERFORMANCE MODELING OF E-PROCUREMENT WORKFLOW USING GENERALISED STOCHASTIC PETRI NET (GSPN)**

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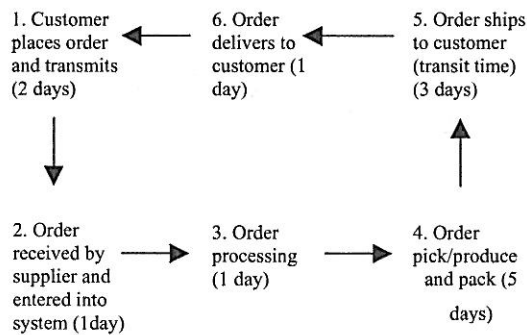
### **ABSTRACT**

This paper proposes a Generalised Stochastic Petri net (GSPN) model representing a generic e-procurement workflow process. The model displays the dynamic behaviour of the system and shows the inter relationship of process activities. An analysis based on matrix equation approach enabled users to analyse the critical system's states, and thus justify the process performance. The results obtained allow users for better decision making in improving e-procurement workflow performance.

**Key words:** Performance modeling, Generalised stochastic petri nets, E-procurement, Workflow.

## 1.0 INTRODUCTION

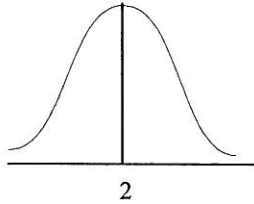
Procurement is the purchase of materials and services from outside organizations to support the firm's operations from production to marketing, sales, and logistics (Lambert *et al.*, 1998). As organizations form longer-term relationships with fewer key suppliers, procurement continues to grow in importance and contribution to the organizations. To maintain the business relationship, both organizations and suppliers must ensure quality. One of the elements of quality is JIT (just-in-time). The JIT focus on the elimination of waste includes the supplier, with the aim of reducing waste and cost throughout the entire supply chain (Lambert *et al.*, 1998). Time base procurement order cycle is one of the JIT elements. With the growth of today's technology, the implementation of JIT becomes possible. The traditional procurement order cycle model adopted from Lambert *et al.* (1998) is as shown in Figure 1.



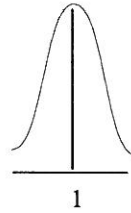
**Fig. 1: Traditional Order Cycle: A Customer's Perspective**

Figure 1 treats the performance of order cycle as if no variability occurred. Figure 2 illustrates the variability that may occur for each component of the order cycle and for the total cycle time. Each of the variable time patterns is assumed to follow a normal statistical distribution.

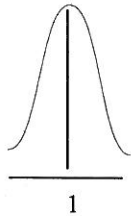
1. Order preparation and transmittal  
Time range 1 to 3 days



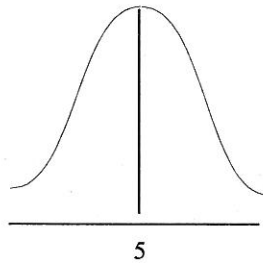
2. Order entry  
- 0.5-1.5 days



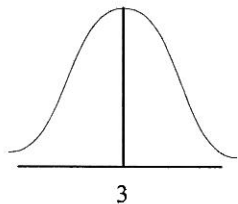
3. Order processing - 0.5 – 1.5 days



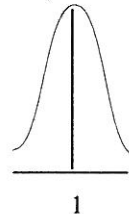
4. Order picking/packing production - 1 – 9 days



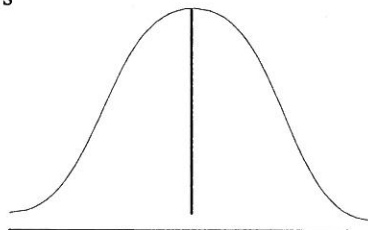
5. Transit time - 1 – 5 days



6. Customer receiving - 0.5 – 1.5 days

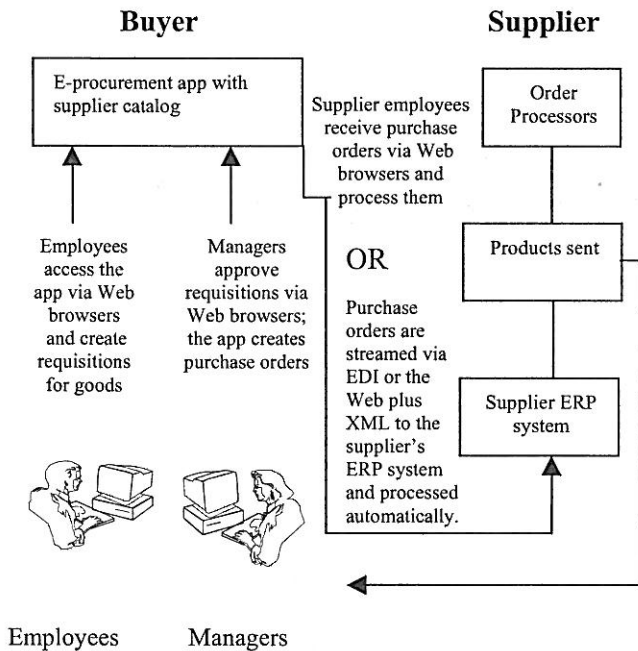


7. Total = 13 days



**Fig. 2: Variability of Order Cycles**

Before the technology explosion, supply chain moves products and information from suppliers to manufacturers to wholesalers/retailers in defined steps. Today information moves independently of the product flows and at the Internet speed. The old supply chains have evolved into networked supply chains. These network supply chains are dynamic, allowing for companies to be included or excluded based upon technological advances, product life cycles, and customer preferences (Kuglin and Rosenbaum, 2001). Significant improvement has been realized from the introduction of e-procurement. Transaction processing cost and time are reduced (Webb, 2001). E-procurement transforms procurement by streamlining and automating each stage of the process from searching for a product or service to ordering, approval, tracking and payment. Figure 3 below shows the e-procurement order cycle on the buy-side.

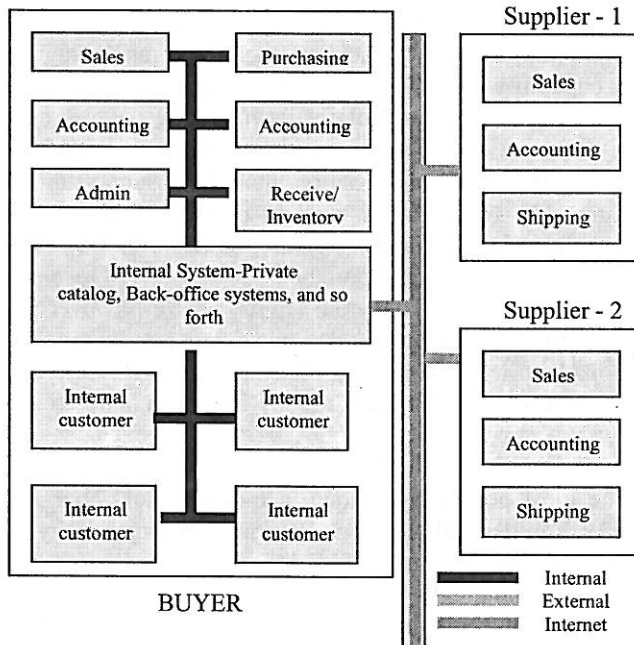


**Fig. 3: E-Procurement Order Cycle: A Buy-Side**  
(Source: *Purchasing Power* by Metz, C., *PC MAGAZINE*, Nov. 2000)

An employee would look at products on a catalog via the organization's intranet. The catalogue contains products/services from approved suppliers and includes information such as specifications, guideline for use and delivery lead-time. Once the requesters know what they want, they fill out an online form and send it via Intranet workflow to their manager. Once approved, the order is

routed to the supplier via the Internet, who can then send back an acknowledgement to the buying organization via the Internet and into the accounting system.

Figure 4 illustrates the integrated interaction between buyer, suppliers and internal customers in e-procurement system adopted from Baron *et al.* (2000).



**Fig. 4: E-Procurement Model: Interaction Between Buyer, Suppliers and Internal Customers.**

Performance measurement provides the means by which a company can assess whether its e-procurement system has improved or degraded. There are traditional methods for measuring supply chains performance such as Balance Scorecard, activity-Based Costing and Economic Value Analysis but the measures do not directly tie to operational effectiveness and efficiency (Lapide, 2000). In the survey relating to the measures of purchasing success, cost and time shows the biggest figure (Segev *et al.*, 1998). The result shows that these two factors represent a big opportunity for process improvement. Response time is one of the principal criteria for the design and implementation of an e-

procurement (Wrigley, 1997). The use of multimedia element in electronic trading may increase the network bandwidth. Communication of browser-server may differ depending on workload which varies depending on the time of day. Hence the hardware and communication architecture must take this range into consideration prior to system implementation. Response time and other performance measures must be tested and run under load conditions. The role of performance modeling is to aid this decision making in an effective way. Schunk and Plott (2000) use discrete event simulation to analyse supply chain processes. Simulation is one of the best means for analyzing supply chains because of its capability for handling variability. However a pitfall in simulation is there is lack of sufficient support for its modeling process and in practice, simulation is never sufficient to prove the correctness of the system (Anu, 1997). Shapiro (2001) use optimisation model in supporting supply chain decision-making. The technique is used to provide managers with effective plans, which are based on company's goals and resource constraint. Business process workflow has been an important issue recently. Queueing model has been used to study workflow behaviour (Enns, 1998); however problem arises when modeling synchronization features and often requires more assumptions and therefore leads to less generally applicable result. A hybrid combination of queueing and simulation (Van Dijk, 2000) has been able to overcome this problem.

Petri net is the most popular technique in modeling and analyzing workflow (Ferscha, 1994); Van der Aalst, 1998 ). Petri nets gain popularity because of its powerful representation. To our knowledge, there is no effort being done to analytically model and analyse e-procurement workflow performance. This research focuses on modeling and analyzing e-procurement workflow performance analytically using Generalised Stochastic Petri net technique (Marsan *et al.*, 1995).

## 2.0 THE GENERALISED STOCHASTIC PETRI NET (GSPN) MODEL

A GSPN is an extension of a Stochastic petri net (SPN). Formally a GSPN is a 6-tuple,  $PN = [P, T, I, O, M_0, -]$  where;  $P = \{p_1, p_2, \dots, p_n\}$  is a finite set of places. A place is a representation of a specific condition/resource.  $T = \{t_1, t_2, \dots, t_n\}$  is a finite set of transitions,  $P \cap T = \phi$  and  $P \cup T = \phi$ . A transition is a representation of a specific action or event. GSPN models comprise two types of transition: timed transition (graphically represented by thick bar), which is associated with random, exponentially distributed firing delays and immediate transition (thin bar), which fires in zero time with firing probabilities.  $F \subseteq (PxT) \cup (TxT)$  is a set of arcs,  $I \in F$  is an input arch and  $O \in F$  is an output

arch.  $M_0: P \rightarrow \{0,1,2,3,\dots\}$  is the initial marking (initial state). A marking or state is an assignment of a nonnegative integer to each place. The number assigned to each place indicates the number of tokens at that place.  $\Lambda: T \rightarrow R$  is a set of firing rates whose entry  $\lambda_k$  is the rate of the exponential individual firing time distribution associated with transition  $t_k$ . The enabling and firing rule for GSPNs are derived as follows:

- i (GSPN enabling rule) A transition  $t \in T$  is enabled in some marking  $M$  at time  $\tau$ , if and only if (iff)  $\forall p \in I(t) \wedge I(t), M(p) \geq w((p,t)$  in  $M$  where  $W: F \rightarrow \{1,2,3,\dots\}$  is a weight function.
- ii (GSPN immediate transition selection rule) Let  $E_\tau(M)$  contain only immediate transitions,  $t_i, t_j \in T$  are said to be in structural conflict, iff  $I(t_i) \cap I(t_j) \neq \emptyset$ .  $t_i, t_j \in T$  are in effective conflict, iff  $t_i, t_j \in E_\tau(M)$  and  $M \xrightarrow{t_i} M'$  might cause that  $t_j \notin E_\tau(M')$ .
- iii (GSPN immediate transition priority rule) Let  $E_\tau(M)$  contain immediate and timed transitions. Then immediate transitions are always selected prior to timed transitions for timing.
- iv (GSPN timed transition race policy) Let  $E_\tau(M)$  contain only timed transitions and let  $RET(t_i), t_i \in E_\tau(u)$  be the remaining enabling time of  $t_i$ .  $RET(t_i)$  is computed from  $X_i$ , the total enabling time, minus the time expired since the enabling instant of  $t_i$ . The transition  $t$  that must fire next is the one with  $t \setminus \min_i RET(t_i)$ .
- v (GSPN firing rule) When a transition  $t \in T$  fires in  $M$  at  $T$ , it creates  $M'$ .

Following the firing and enabling rules for GSPN, an e-procurement GSPN model based on E-procurement process depicted in Figure 2 was developed as shown below. Legends for Figure 5 are shown in Table 1.

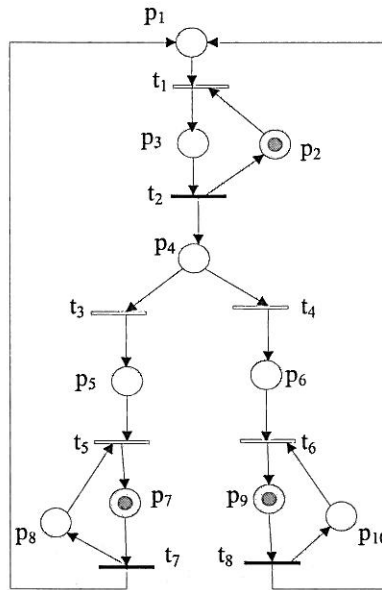


Fig. 5: GSPN E-Procurement Workflow Model

Table 1: Legends for Figure 5

Place	Description
p <sub>1</sub>	Queue of requisition waiting for approval
p <sub>2</sub>	Approval system available
p <sub>3</sub>	Requisition is being processed for approval
p <sub>4</sub>	Approved order
p <sub>5</sub>	Order information in ERP's system queue
p <sub>6</sub>	Order in supplier's system queue
p <sub>7</sub>	Order information being updated
p <sub>8</sub>	ERP's system available
p <sub>9</sub>	Order being processed
p <sub>10</sub>	Supplier's system available



<b>Timed Transition</b>	<b>Description</b>	<b>(<math>\lambda_i</math>)</b>
$t_2$	Approval system ends processing requisition	$\lambda_1$
$t_7$	ERP's system ends updating database	$\lambda_2$
$t_8$	Supplier's system ends processing order	$\lambda_3$
<b>Immediate Transition</b>	<b>Description</b>	
$t_1$	Approval system begins processing requisition	
$t_3$	Order information is entered into ERP system	
$t_4$	Order is transmitted to supplier system	
$t_5$	ERP system begins updating database	
$t_6$	Supplier's system begins processing order	

Figure 6 shows the reachable marking i.e., a marking  $M_j$  is said to be reachable from a marking  $M_0$  if there exists a sequence of firings that transforms  $M_0$  to  $M_j$ .

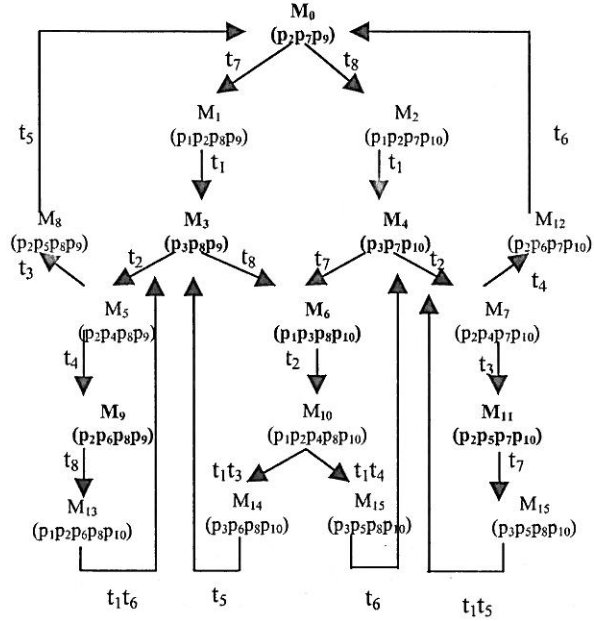


Fig. 6: A Reachable Marking of Figure 5

### 3.0 A COMPUTATIONAL STEADY STATE SOLUTION METHOD

Due to the memoryless properties of the exponential distribution in the stochastic process, the stochastic Petri net is isomorphic to a continuous Markov chain (MC). An analysis method based on identifying embedded Markov chain (EMC) has been used.

Let  $R$  = state space in the process,  $|R| = S_R$

$R_t$  = set of tangible states,  $|R_t| = S_{R_t}$

$R_v$  = set of vanishing states in  $R$ ,  $|R_v| = S_v$  with  $R = R_t \cup R_v$ ,  $R_t \cap R_v = \phi$  and

$S_R = S_t + S_v$

The transition probability of the EMC can be written as follows:

$$U = A + B = \begin{matrix} & \begin{matrix} S_v & S_t \end{matrix} \\ \begin{matrix} S_v \\ S_t \end{matrix} & \begin{bmatrix} C & D \\ 0 & 0 \end{bmatrix} \end{matrix} + \begin{matrix} \begin{matrix} S_v & S_t \end{matrix} \\ \begin{bmatrix} 0 & 0 \\ E & F \end{bmatrix} \end{matrix} \quad (1)$$

The elements of matrix A, represent the probability of the process moving from vanishing states to a vanishing state ( C ) or to a tangible state ( D ) and from tangible states to a vanishing state ( E ) or tangible state ( F ) in matrix B. The transition probability matrix  $U' = [u'_{ij}]$  between tangible states can be computed as follows:

$u'_{ij} = f_{ij} + \sum_{r \in V} e_{ir} \Pr[r \rightarrow j] \quad i, j \in T, r \in V$  written in explicit expression as below

$$U' = F + EG^{\infty}$$

$$\text{where } G^{\infty} = \begin{cases} \left( \sum_{h=0}^{k_0} C^h \right) D & \text{No loops among vanishing states,} \\ [I - C]^{-1} D & \text{Loops among vanishing states,} \end{cases}$$

Let  $Y = (y_1, y_2, \dots, y_n)$  be a vector of real numbers with  $n = s_r$ . Then the solution of the system of linear equations

$$\begin{aligned} Y &= Y.U' \\ \sum_{i=1}^R y_i &= 1 \end{aligned} \tag{3}$$

gives the stationary probabilities of the EMC, where  $y_i$  is the relative number of visits to  $M_i$  by the marking process. Steady state probabilities of the marking process can be obtained using the expression

$$\pi_i = \frac{y_i m_i}{\sum_{j=1}^R y_j m_j} \tag{4}$$

and  $m_i$  is the mean sojourn time of the marking  $M_i$ , which is given by

$$m_i = \frac{1}{\sum_{t_k \in E(M_i)} \lambda_k}$$

#### 4.0 NUMERICAL RESULT

The result of the above computation is deduced by assuming the values of transition firing rate  $\lambda_i$  and the random switch,  $q = 0.4$ . By substituting the values into (1) and compute equations (2), (3) and (4), the steady state probabilities of tangible states are obtained as shown below.

$$\begin{pmatrix} \pi_0 \\ \pi_3 \\ \pi_4 \\ \pi_6 \\ \pi_9 \\ \pi_{11} \end{pmatrix} = \begin{pmatrix} 0.09, 0.101, 0.093 \\ 0.22, 0.1920, 0.234 \\ 0.14, 0.169, 0.118 \\ 0.36, 0.32, 0.295 \\ 0.13, 0.1152, 0.2125 \\ 0.06, 0.1024, 0.0472 \end{pmatrix}$$

The steady-state probability in the first column is derived using  $\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1$  and  $\lambda_1 = 2, \lambda_2 = 1, \lambda_3 = 2$ ,  $\lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 1$ , for the respective columns. From the steady state probability, some performance metric of the system can be calculated. They are depicted in Table 2.

Table 2: E-Procurement Performance Analysis

Performance Indicator	$\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1$	$\lambda_1 = 2, \lambda_2 = 1, \lambda_3 = 2$	$\lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 1$
A1	64%.	68%	70%
A2	1.08	1.001	0.942
A3	0.13	0.1152	0.2125
A4	0.06	0.1024	0.0472
A5	1.45	2.382	2.808
A6	0.69	0.42	0.36
A7	1.165	0.096	0.140

- A1: Utilisation of the system's site  
A2: Average waiting time before requisition approval (time unit)  
A3: Mean queue length of supplier's system (order per unit)  
A4: Mean queue length of ERP system (order per unit)  
A5: Average process throughput rate (order per unit)

- A6: Average cycle time per order (order per unit)  
A7: Waiting time (order per unit)

Results shown in the table indicate that by increasing  $\lambda_1$  and  $\lambda_3$  all performance indicators are improved except for mean queue length of ERP system. The process performance shows a better result when  $\lambda_1$  is increased by one unit and the value of  $\lambda_3$  is reduced by one unit. However mean queue length of supplier's system shows a slightly insignificant result.

Results obtained from the analytical analysis are useful in making the right decision. Process designers are able to determine bottleneck in the process operation. Number of stations required and their specification to meet the target response time can also be identified. The impact of interstation buffering on system performance can be adjusted by reducing/adding or scheduling the buffer accordingly. On the whole, process designers are able to enhance the e-procurement process performance by giving such quantitative measurement values.

## 5.0 CONCLUSION

Although modeling of e-procurement process has been receiving a lot of attention recently, hardly any studies showing the use of quantitative analysis to support the performance of such process have been presented. In this paper we demonstrate the applicability of quantitative modeling and analysis technique from Generalised Stochastic Petri net to model e-procurement workflow. The model is then used to derive performance measures based on embedded Markov chain analysis. These measurements enable e-procurement engineers to analyse the structure and behaviour of an e-procurement system.

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