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A NOVEL EXTENSION OF THE RAYLEIGH DISTRIBUTION FOR BIOMEDICAL DATA: PROPERTIES, ESTIMATION AND PERFORMANCE

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ABSTRACT

This study introduces a new compound distribution by combining the Rayleigh distribution with the sine type II Topp-Leone (STIITL) generator, resulting in the sine type II Topp-Leone Rayleigh (STIITLR) distribution. The proposed model incorporates a trigonometric function and an additional shape parameter, offering greater flexibility in modelling diverse data behaviours. Several statistical properties of the distribution are derived, including moments, the moment-generating function, hazard and survival functions, and entropy. To estimate the model parameters, four methods were employed: maximum likelihood, Anderson-Darling, Cramér-von Mises, and maximum product of spacings. A simulation study was conducted to evaluate the consistency and accuracy of these estimators. As sample sizes increased, the estimators showed improved performance, with reductions in bias and root-mean-square error. The STIITLR model was then applied to a biomedical dataset involving survival times of patients with acute myeloid leukaemia. Compared to competing distributions, the proposed model provided a superior fit. These findings demonstrate that the STIITLR distribution is a promising tool for modelling complex survival patterns in biomedical research, where flexibility and adaptability are essential.

Keywords: Maximum Likelihood Estimation Method, moment, Rayleigh Distribution, Sine Type II Topp-Leone G, survival data.

INTRODUCTION

Statistical distributions are fundamental tools for describing real-world phenomena. Their versatility and ability to model randomness make them crucial across a range of fields, including engineering, finance, environmental science, and ecology. The strength of these distributions lies in their ability to capture patterns and variations in complex data, allowing for predictions and inferences across a wide range of applications. However, as data volumes and complexity grow, classical distributions often fail to capture the full range of data behaviours, particularly when skewness, kurtosis, or multimodality is pronounced. Emerging datasets frequently exhibit characteristics such as heavy tails, asymmetric patterns, or peaks that traditional distributions cannot adequately capture. Consequently, researchers have introduced generalized forms of classical distributions, incorporating additional parameters or mathematical modifications to enhance their flexibility. By compounding classical distributions with generalized forms, these approaches improve their adaptability to diverse data structures, enabling better fits and more accurate representations of real-life data.

Several notable generalized distributions have been introduced in the literature. For instance, Al-Shomrani et al. (2016) proposed the Topp Leone generalized family. Brito et al. (2017) introduced the Topp-Leone Odd Log-Logistic family. On the other hand, Yousof et al. (2018) developed the Marshal Olkin generalized family. Bashiru et al. (2025) introduced the cosine-inverse Lomax-G distribution family. In this research, we focus on the Sine Type II Topp-Leone (STIITL) generalized family proposed by Isa et al. (2023), which combines the Type II Topp-Leone family with the Sine-G family. This combination provides greater flexibility than traditional distributions, thanks to the inclusion of a shape parameter and the Sine function.

In this study, the Rayleigh (R) distribution, introduced by Rayleigh (1880), is employed as the baseline model. It is a one-parameter scale distribution that plays a significant role in modelling and analyzing lifetime data across various fields, including reliability analysis, engineering, the physical sciences, and medical imaging.

Due to its importance, the R distribution has been the foundation for numerous extensions. Prominent examples include the Type II Half Logistic R model (Haq et al., 2018), the exponentiated Weibull R distribution (Elgarhy et al., 2019), the exponentiated Gompertz inverse R distribution (Bashiru et al., 2024), the power transmuted inverse R distribution (Hassan et al., 2020), the Gompertz inverse R model (Halid & Bashiru, 2022), the Topp-Leone Gompertz inverse R distribution (Bashiru & Halid, 2023), the Generalized R distribution (Bashiru et al., 2023), the Topp-Leone exponentiated Gompertz inverse R model (Bashiru et al., 2024), the new extended R distribution (Anabike et al., 2024), and the exponentiated generalized Weibull R distribution (Alsulami, 2025). While these models offer enhanced flexibility, none can universally capture the diverse characteristics of all datasets, highlighting the ongoing need for further distributional development.

The motivation for this research is to address the limitations of the R distribution in tail and peak flexibility. While various extensions have been proposed, many still lack sufficient adaptability. Therefore, our goal is to introduce a new extension of the R model, termed the Sine Type II Topp-Leone Rayleigh (STIITLR) distribution, which will be achieved by merging the STIITL family with the classical R model. This novel distribution aims to provide improved flexibility and enhanced goodness of fit for the R distribution.

The STIITLR model offers several notable advantages, making it a robust choice for modelling lifetime data:

1. The density function exhibits a right-skewed shape, which is often observed in real-world data. This characteristic allows better modelling of phenomena in which extreme values are more likely to occur on one side, thereby enhancing the model's relevance to practical applications.

2. Both the cumulative distribution function (CDF) and quantile function are available in closed forms. This feature simplifies calculations, supporting ease of use in theoretical analysis and practical applications, including statistical inference and data simulation.
3. The hazard function exhibits a monotonically increasing shape, making it particularly suitable for datasets in which the risk of failure or occurrence increases over time. This flexibility is essential for accurately representing a range of real-world scenarios, including reliability engineering and medical research.
4. The model's parameters can be efficiently estimated using four distinct estimation techniques.
5. A simulation study assessed the performance of these estimators. Results showed that bias and mean square error (MS) decrease with increasing sample size, confirming the reliability and consistency of the estimators.
6. The model's efficacy is demonstrated through its application to a real-life biomedical dataset, where it outperforms other extensions of the Rayleigh distribution. This superior performance highlights the model's practical utility and reinforces its value as a robust tool for researchers and practitioners in the field.

FORMULATION OF THE PROPOSED MODEL

The CDF of the STIITL generator is given in Equation 1.

$$F(x) = \sin \left\{ \frac{\pi}{2} \left[1 - (1 - (V(x))^2)^\alpha \right] \right\} \quad (1)$$

Correspondingly, the probability density function (PDF) associated with this generator is expressed as in Equation 2.

$$f(x) = \frac{\pi}{2} 2\alpha v(x)V(x) \left[1 - (V(x))^2 \right]^{\alpha-1} \cos \left\{ \frac{\pi}{2} \left[1 - (1 - (V(x))^2)^\alpha \right] \right\} \quad (2)$$

where α is the shape parameter, $v(x)$ and $V(x)$ are the PDF and CDF of any baseline distribution.

The CDF and PDF of the baseline (R) distribution are given by Equations 3 and 4, respectively.

$$V(x) = 1 - e^{-x^2/(2\theta^2)}, x > 0 \quad (3)$$

$$v(x) = \frac{x}{\theta^2} e^{-x^2/(2\theta^2)}, x > 0 \quad (4)$$

By substituting Equation 3 and Equation 4 into Equations 1 and 2, respectively, the CDF of the STITLR distribution is obtained as in Equation 5.

$$F(x) = \sin \left\{ \frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right)^\alpha \right] \right\}, x > 0 \quad (5)$$

and the corresponding PDF is given by Equation 6.

$$f(x) = \frac{\pi}{2} 2\alpha \left(\frac{x}{\theta^2} e^{-x^2/(2\theta^2)} \right) \left(1 - e^{-x^2/(2\theta^2)} \right) \left[1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right]^{\alpha-1} \cos \left\{ \frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right)^\alpha \right] \right\} \quad (6)$$

$, x, \alpha, \theta > 0$

where α is the shape parameter, and θ is the scale parameter.

The survival function (SF) represents the probability that a subject or component will last beyond a given time, providing insight into its long-term reliability and expected lifespan. It is particularly useful in reliability engineering, healthcare, and risk analysis, where predicting component longevity or survival rates is crucial. For the proposed model, the SF is expressed as in Equation 7.

$$SF = 1 - \sin \left\{ \frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right)^\alpha \right] \right\} \quad (7)$$

The hazard function, HF, provides insight into the instantaneous risk of an event, such as failure, occurring at a given moment, given that the subject has survived up to that time. This function is valuable in identifying when the risk of failure is highest or lowest, offering a detailed view of the risk pattern over time. It finds applications in fields like reliability engineering, medicine, and risk management, where understanding the timing of risk can improve planning, intervention strategies, and resource allocation. For the proposed model, the HF is expressed by Equation 8.

$$HF = \frac{\frac{\pi}{2} 2\alpha \left(\frac{x}{\theta^2} e^{-x^2/(2\theta^2)} \right) \left(1 - e^{-x^2/(2\theta^2)} \right) \left[1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right]^{\alpha-1} \cos \left\{ \frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right)^\alpha \right] \right\}}{1 - \sin \left\{ \frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right)^\alpha \right] \right\}} \quad (8)$$

The reversed hazard (rh) rate, defined as the ratio of the PDF to the CDF, has recently attracted greater interest among researchers. As a complementary counterpart to the traditional hazard rate, the reversed hazard rate reveals unique characteristics that are beneficial for reliability and survival analysis. It provides insights into the likelihood of an event occurring beyond a given time point, offering an alternative perspective on time-to-event data. This function is particularly valuable for assessing the remaining life expectancy of components and systems in reliability studies. The rh function for the proposed model is given by Equation 9.

$$r(x) = \frac{\frac{\pi}{2} 2\alpha \left(\frac{x}{\theta^2} e^{-x^2/(2\theta^2)} \right) \left(1 - e^{-x^2/(2\theta^2)} \right) \left[1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right]^{\alpha-1} \cos \left\{ \frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right)^\alpha \right] \right\}}{\sin \left\{ \frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x^2/(2\theta^2)} \right)^2 \right)^\alpha \right] \right\}} \quad (9)$$

The quantile function (QF) provides a comprehensive description of a random variable’s statistical properties by linking specific probability levels with their corresponding data values, effectively capturing the distribution's shape across its entire range. This function is especially useful for identifying threshold values for different probability levels, enabling detailed probability-based interpretations, such as risk assessment and prediction intervals. Additionally, QFs are instrumental in fields like finance, quality control, and reliability engineering, where precise thresholds can guide decision-making under uncertainty. For the proposed model, the QF is given by Equation 10.

$$QF = \left[\left(2\theta^2 \right) \left(-\ln \left(1 - \left(1 - \left(1 - \frac{\sin^{-1}(u)}{\frac{\pi}{2}} \right)^{\frac{1}{\alpha}} \right)^{\frac{1}{2}} \right) \right) \right]^{\frac{1}{2}} \quad (10)$$

The plots in Figures 1 to 4 provide valuable insights into the characteristics of the STITLR distribution. From Figure 1, the PDF plot of the STITLR distribution exhibits positive skewness, indicating a right-tailed distribution. Figure 2 presents the CDF, which spans the range from 0 to 1, confirming that the STITLR distribution is a valid probability model, as a CDF must approach 1 as it approaches the upper limit of its domain. In Figure 3, the hazard function plot shows an increasing failure rate, suggesting that the STITLR distribution is particularly suited to modelling data with a likelihood of failure that increases over time. This characteristic is essential for reliability and survival analysis, as it aligns with datasets that exhibit wear-out failure modes common in engineering components, medical survival times, and similar applications where risk increases with age or usage. Finally, Figure 4 displays the survival function, which decreases steadily from 1 to 0. This behaviour represents the probability of survival or reliability declining over time, affirming that the STITLR distribution can effectively capture the decay in survival probability as time progresses. This makes it a strong candidate for datasets where the probability of survival diminishes consistently, such as in clinical studies, mechanical part lifetimes, and other time-to-event analyses.

Figure 1

PDF Plot of STITLR Distribution

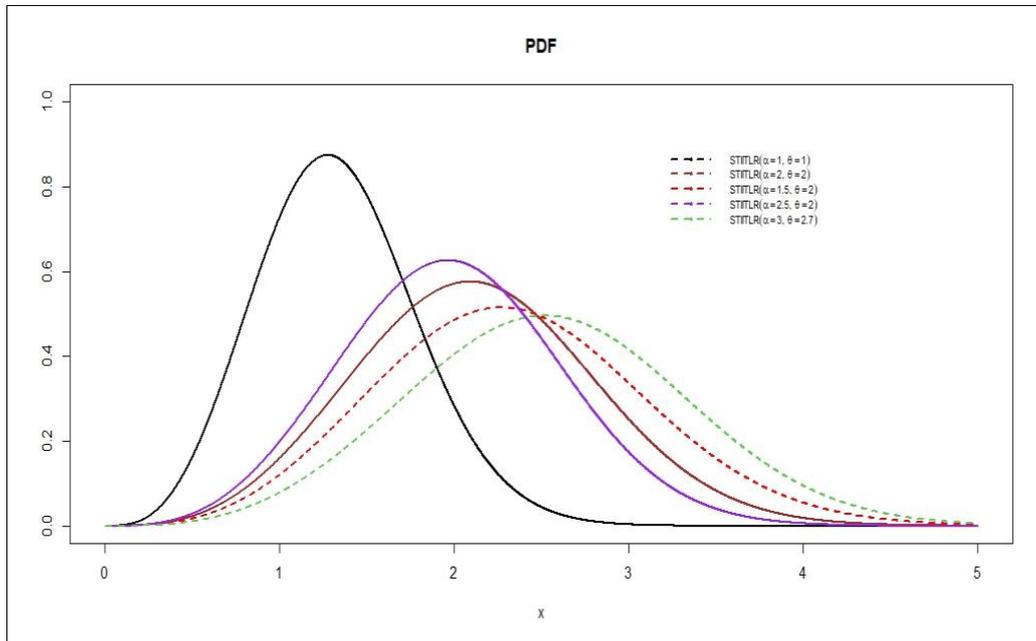


Figure 2

CDF Plot of STITLR Distribution

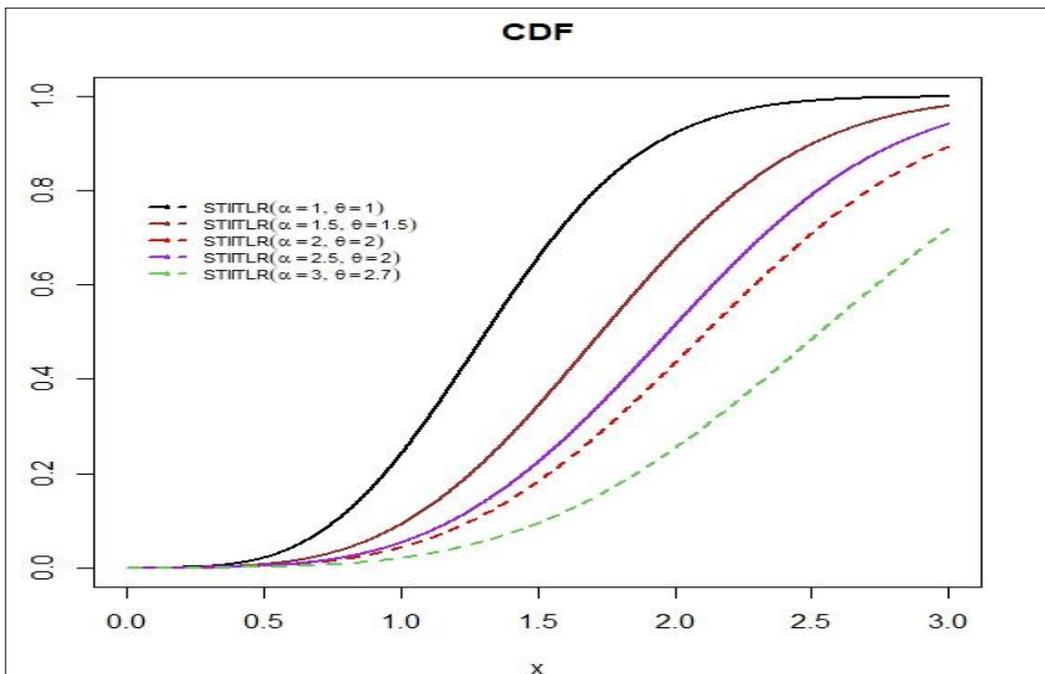


Figure 3

Hazard Plot of STITLR Distribution

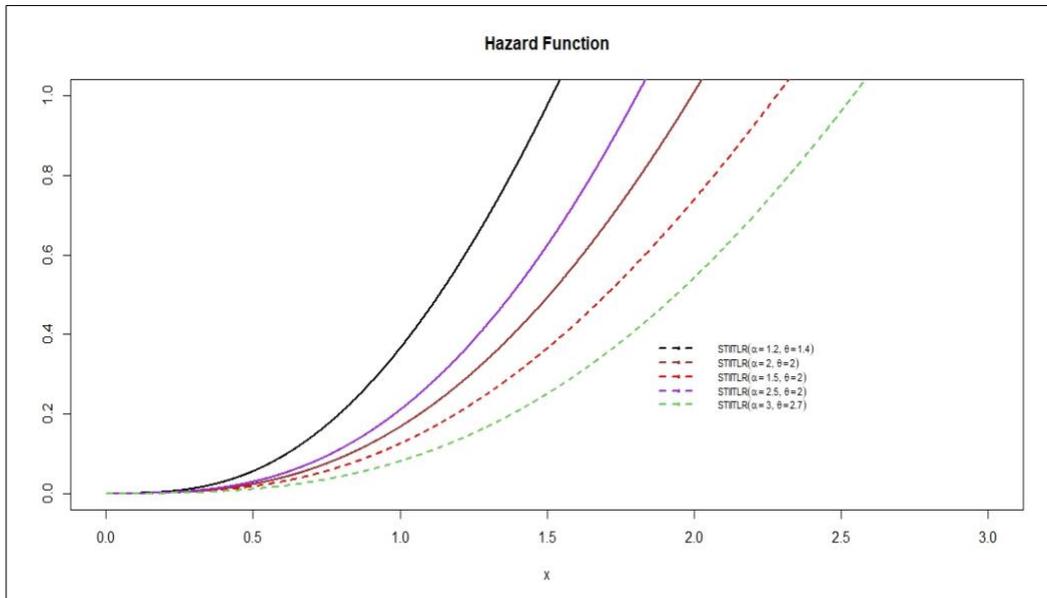
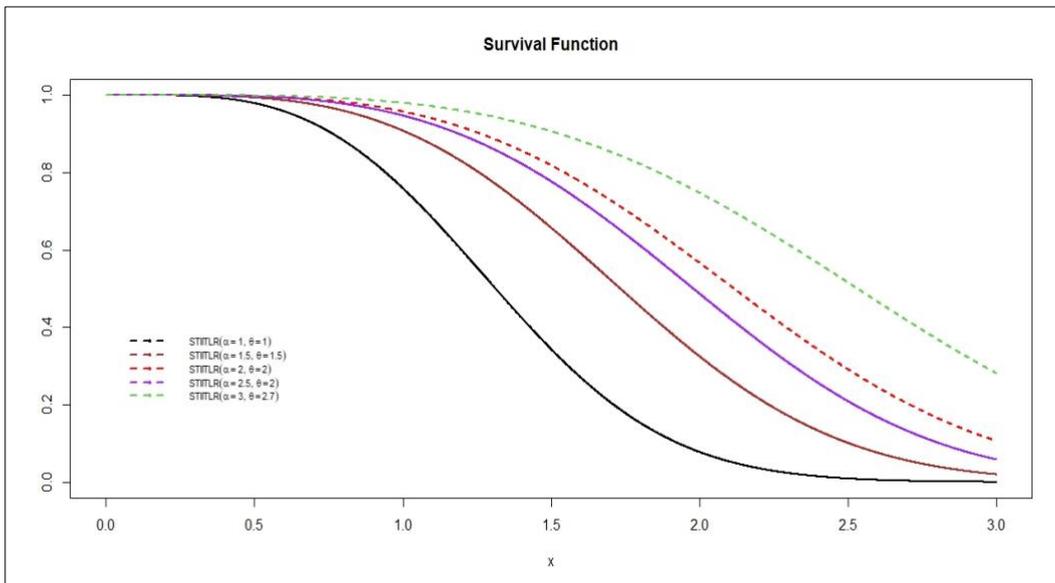


Figure 4

Survival Function Plot of STITLR Distribution



MATHEMATICAL PROPERTIES

Linear Expansion of the PDF

The PDF of the STIITLR distribution, as given in Equation 6, can be expressed in an expanded form using a power series. Specifically, the Taylor series expansions for the sine and cosine functions are given as in Equations 11 and 12.

$$\cos(x) = \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i}}{2i!}, \tag{11}$$

$$\sin(x) = \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i+1}}{(2i+1)!} \tag{12}$$

Then, employing the expansion in Equation 11 for the last term of the PDF in Equation 6 yields Equation 13.

$$\cos \left\{ \frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}} \right)^2 \right)^\alpha \right] \right\} = \sum_{i=0}^{\infty} \frac{(-1)^i \pi^{2i}}{(2i)! 2^{2i}} \left[1 - \left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}} \right)^2 \right)^\alpha \right]^{2i} \tag{13}$$

Furthermore, applying the binomial expansion to Equation 13 yields Equation 14.

$$\left[1 - \left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}} \right)^2 \right)^\alpha \right]^{2i} = \sum_{j=0}^{2i} (-1)^j \binom{2i}{j} \left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}} \right)^2 \right)^{\alpha j} \tag{14}$$

Equation 6 becomes Equation 15.

$$f(x) = \frac{\pi}{2} 2\alpha \left(\frac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}} \right) \left(1 - e^{-\frac{x^2}{2\theta^2}} \right) \sum_{i=0}^{\infty} \sum_{j=0}^{2i} \frac{(-1)^{i+j}}{(2i)!} \binom{2i}{j} \left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}} \right)^2 \right)^{\alpha j + \alpha - 1} \tag{15}$$

Again, using binomial expansion, we have:

$$\left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}} \right)^2 \right)^{\alpha j + \alpha - 1} = \sum_{k=0}^{\infty} (-1)^k \binom{\alpha j + \alpha - 1}{k} \left(1 - e^{-\frac{x^2}{2\theta^2}} \right)^{2k}$$

The term $\left(1 - e^{-\frac{x^2}{2\theta^2}}\right)$ can further be expanded using the binomial series as follows:

$$\left(1 - e^{-\frac{x^2}{2\theta^2}}\right)^{2k+1} = \sum_{l=0}^{2k+1} (-1)^l \binom{2k+1}{l} e^{-\frac{x^2 l}{2\theta^2}}$$

Therefore, Equation 6 becomes

$$f(x) = \sum_{i,k=0}^{\infty} \sum_{j=0}^{\infty} \sum_{l=0}^{2k+1} \frac{(-1)^{i+j+k+l}}{(2i)!} \frac{\pi^{2i+1}}{2^{2i+1}} \binom{2i}{j} \binom{\alpha j + \alpha - 1}{k} \binom{2k+1}{l} \frac{2\alpha x}{\theta^2} e^{-\frac{x^2 l}{2\theta^2}},$$

Hence, the PDF can be expressed by Equation 16.

$$f(x) = \sum_{i,k=0}^{\infty} \phi_{i,j,k,l} \frac{2\alpha x}{\theta^2} e^{-\frac{x^2 l}{2\theta^2}} \tag{16}$$

where,

$$\phi_{i,j,k,l} = \sum_{i,k=0}^{\infty} \sum_{j=0}^{\infty} \sum_{l=0}^{2k+1} \frac{(-1)^{i+j+k+l}}{(2i)!} \frac{\pi^{2i+1}}{2^{2i+1}} \binom{2i}{j} \binom{\alpha j + \alpha - 1}{k} \binom{2k+1}{l}$$

Moments

Moments are key statistical measures that provide essential insights into the characteristics and shape of a probability distribution. The r^{th} moment of a random variable conveys various properties of its distribution: the first moment indicates the mean, the second moment relates to variance, and higher-order moments provide information about skewness and kurtosis. In practical applications involving real-world datasets, moments are instrumental for summarizing and interpreting data behaviour. For instance, the mean reflects the central tendency, while variance measures variability. Skewness indicates asymmetry within the distribution, and kurtosis highlights the presence of outliers or the distribution's tail behaviour. By examining these moments, analysts and researchers can gain a deeper understanding of data patterns, which can inform decision-making across various domains, including finance, healthcare, and engineering.

Let X be a random variable following the STITLR distribution. The r^{th} moment of X is given by Equation 17.

$$\mu_r = E(x^r) = \int_{-\infty}^{\infty} x^r f(x) dx \tag{17}$$

By substituting the PDF from Equation 16 into Equation 17, the expression for the r^{th} moment is obtained as:

$$\begin{aligned} \mu_r &= \sum_{i,k=0}^{\infty} \phi_{i,j,k,l} \frac{2\alpha}{\theta^2} \int_0^{\infty} x^{r+1} e^{-\frac{x^2 l}{2\theta^2}} dx \\ &= \sum_{i,k=0}^{\infty} \phi_{i,j,k,l} \frac{2\alpha}{\theta^2} \frac{2^{r/2} \theta^{r+2}}{l^{(r+2)/2}}. \end{aligned}$$

The first four moments of the distribution, namely the mean, second moment, third moment, and fourth moment, can then be determined by setting $r = 1, 2, 3$ and 4 in Equation 17, respectively.

Moments Generating Function (MGF)

The MGF is a powerful tool for encapsulating all moments of a random variable in a single function. It provides a concise way to derive the mean, variance, and higher-order moments of a distribution. By taking the derivatives of the MGF at zero, one can obtain the moments of the distribution, making it an essential component in probability theory and statistics. The MGF is particularly useful for characterizing the distributions of independent random variables and simplifying calculations involving sums of independent random variables, as it facilitates the analysis of their joint distributions.

Let X be a random variable following the STITLR distribution. The MGF of X is given by Equation 18.

$$M_x(t) = \sum_{i,k=0}^{\infty} \phi_{i,j,k,l} \frac{2\alpha}{\theta^2} \int_0^{\infty} x e^{tx} e^{-\frac{x^2 l}{2\theta^2}} dx. \tag{18}$$

Renyi Entropy (RE)

Entropy is a fundamental concept in information theory that quantifies the uncertainty or unpredictability associated with a random variable. It measures the average amount of information produced by a stochastic process, reflecting the level of disorder within a dataset. In the context of probability distributions, entropy provides insight into the distribution's spread and variability; a higher entropy value indicates a greater level of uncertainty, while lower values suggest more predictability. Entropy is particularly useful in various applications, including data compression, statistical inference, and machine learning, where understanding the underlying uncertainty in data can lead to better models and decision-making. Among the various measures of entropy, the Rényi entropy (RE) provides a generalized framework that extends the traditional Shannon entropy by introducing a parameter δ , which controls the order of the entropy.

The RE formula is given by Equation 19.

$$I_{\delta}(x, \theta) = \frac{1}{1-\delta} \log \int_{-\infty}^{\infty} f(x; \theta)^{\delta} dx \tag{19}$$

For the STITLR distribution, the RE is given by Equation 20.

$$I_{\delta}(x, \theta) = \frac{1}{1-\delta} \log \int_0^{\infty} \left(\sum_{i,k=0}^{\infty} \phi_{i,j,k,l} \frac{2\alpha x}{\theta^2} e^{-\frac{x^2 l}{2\theta^2}} \right)^{\delta} dx \quad (20)$$

PARAMETER ESTIMATION

Maximum Likelihood (ML) Method

The maximum likelihood (ML) method is a widely used and powerful technique for estimating the parameters of statistical distributions. It identifies the parameter values that maximize the likelihood of observing the given data under the assumed model. This method revolves around the likelihood function, which expresses the probability of the observed data as a function of the parameters. By optimizing this function, the ML method yields parameter estimates that best align the model with the data. For the proposed STITLR distribution, the ML method is employed to estimate its parameters. The likelihood function corresponding to the PDF in equation (6) is given by Equation 21.

$$\begin{aligned} \ell = & n \log\left(\frac{\pi}{2}\right) + n \log(2) + n \log(\alpha) + \frac{1}{\theta^2} \sum_{i=1}^n x_i - \frac{1}{2\theta^2} \sum_{i=1}^n x_i^2 + \sum_{i=1}^n \log\left(1 - e^{-x_i^2/(2\theta^2)}\right) \\ & + (\alpha - 1) \sum_{i=1}^n \log\left(1 - \left(1 - e^{-x_i^2/(2\theta^2)}\right)^2\right) + \sum_{i=1}^n \log \cos \left\{ \frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x_i^2/(2\theta^2)}\right)^2\right)^{\alpha} \right] \right\} \end{aligned} \quad (21)$$

Taking the partial derivative of the log-likelihood function with respect to α yields Equation 22.

$$\frac{d\ell}{d\alpha} = \frac{n}{\alpha} + \sum_{i=1}^n \log\left(1 - \left(1 - e^{-x_i^2/(2\theta^2)}\right)^2\right) + \sum_{i=1}^n \tan\left(\frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x_i^2/(2\theta^2)}\right)^2\right)^{\alpha} \right]\right) \frac{\pi}{2} \left(1 - \left(1 - e^{-x_i^2/(2\theta^2)}\right)^2\right)^{\alpha} \log\left(1 - \left(1 - e^{-x_i^2/(2\theta^2)}\right)^2\right) \quad (22)$$

Similarly, differentiating the log-likelihood function with respect to θ gives Equation 23.

$$\begin{aligned} \frac{d\ell}{d\theta} = & -\frac{2}{\theta^3} \sum_{i=1}^n x_i + \frac{1}{\theta^3} \sum_{i=1}^n x_i^2 + \sum_{i=1}^n \frac{e^{-x_i^2/(2\theta^2)} x_i^2}{\theta^3 (1 - e^{-x_i^2/(2\theta^2)})} + (\alpha - 1) \sum_{i=1}^n \frac{2(1 - e^{-x_i^2/(2\theta^2)}) e^{-x_i^2/(2\theta^2)} x_i^2}{\theta^3 (1 - (1 - e^{-x_i^2/(2\theta^2)})^2)} \\ & - \sum_{i=1}^n \tan\left(\frac{\pi}{2} \left[1 - \left(1 - \left(1 - e^{-x_i^2/(2\theta^2)}\right)^2\right)^\alpha\right]\right) \frac{\pi}{2} \alpha \left(1 - \left(1 - e^{-x_i^2/(2\theta^2)}\right)^2\right)^{\alpha-1} \frac{2(1 - e^{-x_i^2/(2\theta^2)}) e^{-x_i^2/(2\theta^2)} x_i^2}{\theta^3} \end{aligned} \tag{23}$$

Equations 22 and 23 are the ML estimates of the STITLR model's parameters.

Method of Maximum Product of Spacings

An effective alternative to the maximum likelihood estimation (MLE) method is the maximum product of spacings (MPS) technique. This approach, referred to as maximum product of spacings estimation (MPSE), involves maximizing the product of the spacings between the ordered data points. For the proposed model, the MPS estimates of the parameters are obtained by maximizing the following function:

$$MPS = \frac{1}{n+1} \sum_{i=1}^{n+1} \log I_i(x_i)$$

where:

$$I_i(x_i) = F(x_{i:n}) - F(x_{i-1:n})$$

$$F(x_{0:n}) = 0 \text{ and } F(x_{n+1:n}) = 1.$$

Method of Anderson-Darling

The Anderson-Darling (AD) method is a widely used estimation technique based on goodness-of-fit principles. It places more emphasis on discrepancies in the tails of the distribution by comparing the empirical and theoretical cumulative distribution functions (CDFs). This characteristic makes it particularly useful in applications where tail behaviour is of interest. For the proposed distribution, the AD estimates of the parameters are obtained by minimising the following function:

$$A(x_i) = -n - \frac{1}{n} \sum_{i=1}^n (2i-1) [\log F(x_{i:n}) + \log S(x_{n-i-1:n})].$$

Method of Cramer-von Mises

The Cramér–von Mises (CVM) estimation method determines the parameters of a theoretical distribution by minimizing the CVM statistic, which quantifies the discrepancy between the empirical and theoretical CDFs. For the STITLR model, the CVM estimates of the parameters are obtained by minimizing the following function:

$$C(x_i) = \frac{1}{12n} + \sum_{i=1}^n \left[F(x_{i:n}) - \frac{2i-1}{2n} \right]^2.$$

SIMULATION STUDY

To evaluate the performance of the proposed STITLR model, a comprehensive Monte Carlo simulation study was conducted. The primary objective was to assess the accuracy and efficiency of four prominent estimation methods: Maximum Likelihood Estimation (MLE), Maximum Product of Spacings (MPS), Anderson-Darling (AD), and Cramer-von Mises (CVM). We specifically examined how well these methods estimate the model parameters by computing their mean estimates (M), biases (B), and mean square errors (MS). Simulated datasets were generated according to the expression in Equation 10. Various sample sizes were used to investigate the behaviour of the estimators across different data volumes. Each scenario was replicated 5,000 times to ensure the statistical robustness of the results. To comprehensively test the model's performance under various conditions, different combinations of true parameter values (α and θ) were employed for the data generation. The specific parameter combinations and corresponding estimation results, including their biases and MSs, are meticulously summarized in Tables 1 to 6 below. The chosen parameter values for α and θ were selected to reflect a diverse range of scenarios where the STITLR model might be applied. A critical characteristic of the STITLR model, like many statistical distributions, is that its parameters, α and θ , are typically defined as positive values (greater than zero). This positivity ensures the mathematical validity and meaningful interpretation of the model's probability density function, cumulative distribution function, and other properties. By selecting parameter values that are consistently greater than zero, we adhere to the intrinsic nature of the STITLR model and investigate its performance within its valid parameter space. Table 1 to 6 present the mean estimates (M), biases (B), and mean square errors (MS) for α and θ across the different estimation methods and sample sizes.

Table 1

M, B and MS of the STITLR Distribution $\alpha=1.5, \theta=2.5$

<i>n</i>	Properties	MLE		MPS		AD		CVM	
		α	θ	α	θ	α	θ	α	θ
20	M	2.4225	2.6401	1.4293	2.0752	2.0547	2.4606	2.2989	2.5721
	B	1.6118	0.7810	1.3084	0.9152	1.4902	0.8269	1.6236	0.8380
	MS	3.3679	0.7794	2.2729	1.1033	2.9147	0.8733	3.3525	0.8790
50	M	2.1884	2.6255	1.4927	2.2725	1.9781	2.5214	2.1682	2.5931
	B	1.3024	0.6207	1.0351	0.6388	1.2441	0.6414	1.3813	0.6776
	MS	2.5032	0.5099	1.6600	0.5740	2.2750	0.5457	2.6861	0.5893
70	M	2.1439	2.6366	1.5111	2.3233	1.9495	2.5377	2.1218	2.5944
	B	1.1962	0.5617	0.9307	0.5541	1.1479	0.5810	1.2919	0.6228
	MS	2.2240	0.4282	1.4555	0.4486	2.0253	0.4571	2.4547	0.5087
100	M	2.0566	2.6235	1.5092	2.3627	1.9119	2.5505	2.0365	2.5875
	B	1.0755	0.5065	0.8031	0.4690	1.0360	0.5150	1.1537	0.5541
	MS	1.8838	0.3562	1.1907	0.3425	1.7428	0.3703	2.0810	0.4178
150	M	1.9422	2.6010	1.5010	2.3964	1.8647	2.5541	1.9799	2.5890
	B	0.9165	0.4363	0.6566	0.3787	0.9245	0.4551	1.0398	0.4926
	MS	1.4718	0.2782	0.9050	0.2452	1.4615	0.2983	1.7801	0.3397
250	M	1.8302	2.5832	1.4914	2.4302	1.7850	2.5525	1.8718	2.5767
	B	0.7269	0.3514	0.4720	0.2718	0.7509	0.3712	0.8538	0.4068
	MS	0.9974	0.1919	0.5859	0.1531	1.0301	0.2083	1.3019	0.2463
300	M	1.7871	2.5726	1.5130	2.4494	1.7331	2.5398	1.8355	2.5703
	B	0.6650	0.3251	0.4279	0.2435	0.6806	0.3392	0.7939	0.3805
	MS	0.8496	0.1664	0.5247	0.1317	0.8700	0.1780	1.1551	0.2185
500	M	1.6780	2.5456	1.4872	2.4632	1.6715	2.5360	1.7368	2.5529
	B	0.4997	0.2535	0.2594	0.1532	0.5286	0.2677	0.6211	0.3021
	MS	0.4824	0.1032	0.2746	0.0707	0.5460	0.1156	0.7576	0.1467
1000	M	1.5859	2.5226	1.4876	2.4850	1.5780	2.5134	1.6260	2.5297
	B	0.3332	0.1761	0.0955	0.0629	0.3633	0.1908	0.4269	0.2169
	MS	0.2046	0.0503	0.0781	0.0218	0.2461	0.0592	0.3499	0.0769

Table 2

M, B and MS of the STITLR Distribution $\alpha=0.3, \theta=1.4$

<i>n</i>	Properties	MLE		MPS		AD		CVM	
		α	θ	α	θ	α	θ	α	θ
20	M	1.1067	1.7837	0.5188	1.3409	0.8622	1.6399	1.1375	1.7815
	B	0.9398	0.7731	0.4606	0.5858	0.7281	0.688	0.9874	0.7956
	MS	2.5169	1.001	0.8962	0.5519	1.7763	0.8132	2.8000	1.0781
50	M	0.6285	1.5746	0.3562	1.3184	0.5381	1.5108	0.6836	1.5955
	B	0.4438	0.4966	0.2510	0.4003	0.3743	0.4485	0.5118	0.5219
	MS	0.7217	0.4511	0.2560	0.2582	0.5683	0.3627	1.0242	0.5086
70	M	0.4956	1.5003	0.3281	1.3328	0.4515	1.4715	0.5672	1.5481
	B	0.3002	0.3976	0.1962	0.3368	0.2728	0.3649	0.3843	0.4359
	MS	0.3238	0.2829	0.1171	0.1802	0.2925	0.2409	0.5974	0.3544
100	M	0.4372	1.4641	0.3116	1.3400	0.3970	1.4511	0.4653	1.4943
	B	0.2292	0.3284	0.1588	0.2833	0.2038	0.3027	0.2745	0.3558
	MS	0.1812	0.199	0.0632	0.1278	0.1471	0.1587	0.2975	0.2324
150	M	0.3792	1.4218	0.2979	1.3497	0.3535	1.4232	0.4067	1.4758
	B	0.1632	0.2709	0.1208	0.2249	0.1502	0.2441	0.1989	0.2879
	MS	0.0622	0.1301	0.0312	0.0805	0.0601	0.1003	0.1311	0.1478
250	M	0.3484	1.4075	0.2968	1.3661	0.3283	1.4153	0.3558	1.4474
	B	0.1172	0.2077	0.0926	0.1743	0.1045	0.1819	0.1291	0.2083
	MS	0.0309	0.0838	0.0151	0.0489	0.0210	0.0531	0.0395	0.073

300	M	0.3438	1.4103	0.2989	1.3739	0.3194	1.4037	0.3411	1.4276
	B	0.1081	0.1911	0.0867	0.1603	0.0961	0.1687	0.1174	0.1942
	MS	0.0247	0.0709	0.0146	0.0431	0.0178	0.0455	0.0303	0.0623
500	M	0.3283	1.3907	0.2934	1.3740	0.3149	1.4076	0.3207	1.4163
	B	0.0836	0.1557	0.0631	0.1191	0.0755	0.1354	0.0796	0.1393
	MS	0.0159	0.0543	0.0068	0.0240	0.0099	0.0288	0.0121	0.0319
1000	M	0.3162	1.3651	0.2978	1.3895	0.3082	1.4061	0.3116	1.4103
	B	0.0670	0.1284	0.0445	0.0820	0.0502	0.0914	0.0548	0.0981
	MS	0.0140	0.0511	0.0035	0.0122	0.0044	0.0136	0.0051	0.0152

Table 3

M, B and MS of the STITLR Distribution $\alpha=0.8, \theta=0.5$

n	Properties	MLE		MPS		AD		CVM	
		α	θ	α	θ	α	θ	α	θ
20	M	1.8794	0.5752	1.0541	0.4479	1.5844	0.5368	1.8185	0.5642
	B	1.4450	0.2077	0.9395	0.1982	1.2597	0.2051	1.4458	0.2163
	MS	3.7018	0.0588	1.8211	0.0530	3.0169	0.0576	3.7465	0.0632
50	M	1.4961	0.5552	0.9158	0.4659	1.333	0.5306	1.5408	0.5523
	B	1.0109	0.1537	0.6633	0.1392	0.9163	0.1497	1.1039	0.1671
	MS	2.2331	0.0369	1.0097	0.0283	1.8885	0.0341	2.5397	0.0411
70	M	1.3133	0.5423	0.8983	0.4757	1.2234	0.5251	1.4363	0.5475
	B	0.8023	0.1299	0.5697	0.1186	0.7654	0.1276	0.9671	0.1474
	MS	1.5153	0.0272	0.7715	0.0215	1.4213	0.0263	2.0817	0.0337
100	M	1.1911	0.5332	0.8689	0.4803	1.1302	0.5225	1.2899	0.5362
	B	0.6457	0.1073	0.4826	0.1004	0.6315	0.1094	0.8013	0.1277
	MS	1.0917	0.0199	0.5675	0.0160	0.9882	0.0195	1.5293	0.0261
150	M	1.0486	0.5212	0.8165	0.4828	1.0266	0.5139	1.1999	0.5339
	B	0.4844	0.0880	0.3621	0.0795	0.5007	0.0904	0.6666	0.1089
	MS	0.6027	0.0133	0.3052	0.0103	0.6637	0.0140	1.1235	0.0200
250	M	0.9533	0.5153	0.8053	0.4888	0.9301	0.5095	1.0384	0.5231
	B	0.3411	0.0658	0.2710	0.0609	0.3499	0.0679	0.4499	0.0796
	MS	0.2877	0.0075	0.1548	0.0061	0.2924	0.0078	0.5287	0.0110
300	M	0.9359	0.5146	0.8074	0.4908	0.8949	0.5050	0.9907	0.5160
	B	0.3100	0.0607	0.2542	0.0563	0.3161	0.0628	0.4135	0.0750
	MS	0.2178	0.0064	0.1492	0.0054	0.2249	0.0065	0.4330	0.0097
500	M	0.8703	0.5078	0.7850	0.4908	0.8667	0.5051	0.8961	0.5091
	B	0.2189	0.0457	0.1761	0.0412	0.2445	0.0504	0.2728	0.0539
	MS	0.0910	0.0034	0.0632	0.0029	0.1148	0.0041	0.1681	0.0050
1000	M	0.8277	0.5025	0.7933	0.4957	0.8350	0.5034	0.8498	0.5054
	B	0.1457	0.0315	0.1153	0.0273	0.1592	0.0338	0.1802	0.0374
	MS	0.0380	0.0017	0.0313	0.0015	0.0479	0.0019	0.0610	0.0023

Table 4

M, B and MS of the STITLR Distribution $\alpha=2.5, \theta=3.5$

<i>n</i>	Properties	MLE		MPS		AD		CVM	
		α	θ	α	θ	α	θ	α	θ
20	M	2.5853	3.2765	1.6810	2.724	2.157	3.0680	2.2786	3.1431
	B	1.2844	0.6898	1.3670	1.0246	1.3218	0.8481	1.2884	0.8056
	MS	1.9863	0.8195	2.5958	1.7407	2.1778	1.1803	2.0907	1.0735
50	M	2.5794	3.3715	1.8666	2.9949	2.3225	3.2444	2.4164	3.2841
	B	1.1786	0.5885	1.0931	0.7196	1.2151	0.6721	1.2274	0.6738
	MS	1.6543	0.5113	1.8484	0.8780	1.773	0.6705	1.8101	0.6785
70	M	2.6014	3.4028	1.9760	3.0987	2.405	3.3056	2.4743	3.3358
	B	1.1327	0.5580	0.9443	0.5984	1.1546	0.6044	1.1828	0.6164
	MS	1.5373	0.4464	1.5188	0.6473	1.6159	0.5387	1.6676	0.557
100	M	2.6214	3.4348	2.0855	3.1881	2.4647	3.3604	2.4921	3.3573
	B	1.0567	0.5020	0.7965	0.4873	1.0967	0.5529	1.1554	0.587
	MS	1.3672	0.3541	1.2273	0.4685	1.4519	0.4261	1.5794	0.4825
150	M	2.6219	3.4506	2.1614	3.2657	2.482	3.3873	2.5529	3.4143
	B	0.9983	0.4672	0.6100	0.3732	1.0167	0.4968	1.0704	0.5228
	MS	1.2367	0.2997	0.9018	0.3250	1.2757	0.3446	1.3836	0.3757
250	M	2.6513	3.4926	2.3056	3.3667	2.5546	3.4457	2.6469	3.4791
	B	0.8704	0.3928	0.3745	0.2329	0.9087	0.4220	0.9788	0.4491
	MS	0.9837	0.2077	0.5245	0.1755	1.056	0.2447	1.1907	0.274
300	M	2.6844	3.5120	2.3287	3.3901	2.5052	3.4287	2.5737	3.4471
	B	0.8511	0.3757	0.3197	0.1978	0.8805	0.4080	0.9568	0.4397
	MS	0.9540	0.1917	0.4255	0.1353	0.9933	0.2277	1.1373	0.2598
500	M	2.6630	3.5204	2.4118	3.4475	2.5885	3.4797	2.6033	3.4867
	B	0.7397	0.3221	0.1347	0.0957	0.7982	0.3568	0.8114	0.3625
	MS	0.7565	0.1437	0.1662	0.0539	0.8496	0.1759	0.8789	0.1808
1000	M	2.5998	3.5124	2.4933	3.4973	2.5984	3.5079	2.6436	3.5195
	B	0.5681	0.2470	0.0303	0.0318	0.6076	0.2650	0.6800	0.2925
	MS	0.4881	0.0894	0.0265	0.0072	0.5450	0.1019	0.6594	0.1203

Table 5

M, B and MS of the STITLR Distribution $\alpha=1.0, \theta=1.8$

n	Properties	MLE		MPS		A.D		CVM	
		α	θ	α	θ	α	θ	α	θ
20	M	2.0925	2.0192	1.2089	1.5832	1.7444	1.8743	1.9673	1.9613
	B	1.5493	0.686	1.0778	0.6946	1.3543	0.6931	1.5159	0.7188
	MS	3.7887	0.6127	2.0333	0.6387	3.0563	0.6313	3.6628	0.6669
50	M	1.7677	1.977	1.1109	1.6657	1.5635	1.8836	1.7566	1.9501
	B	1.1696	0.5316	0.8128	0.4992	1.062	0.5248	1.2223	0.568
	MS	2.5711	0.4097	1.2936	0.3523	2.1494	0.3898	2.6945	0.4457
70	M	1.6243	1.9500	1.109	1.7073	1.4775	1.8767	1.6808	1.9443
	B	0.9942	0.4674	0.7103	0.4265	0.9223	0.4559	1.1063	0.5092
	MS	1.9849	0.3279	1.0509	0.2703	1.7322	0.3106	2.3487	0.3763
100	M	1.4884	1.9210	1.0834	1.7247	1.3993	1.8756	1.5443	1.9129
	B	0.8179	0.3910	0.6143	0.3634	0.7925	0.3973	0.9463	0.448
	MS	1.4854	0.2447	0.8366	0.2059	1.3371	0.2418	1.8359	0.3002
150	M	1.3399	1.8847	1.0249	1.7362	1.2839	1.8487	1.4744	1.9144
	B	0.6436	0.3275	0.4671	0.2879	0.6415	0.3314	0.8187	0.3886
	MS	0.9452	0.1753	0.4897	0.1364	0.9473	0.1773	1.4766	0.2407
250	M	1.2219	1.8628	1.016	1.759	1.1801	1.8369	1.3080	1.8843
	B	0.4668	0.2491	0.3544	0.2197	0.4666	0.2522	0.5849	0.2937
	MS	0.5105	0.1059	0.2932	0.0859	0.5024	0.1066	0.7871	0.1426
300	M	1.2002	1.8611	1.0159	1.7672	1.1318	1.8192	1.2556	1.8591
	B	0.4261	0.2291	0.3243	0.2001	0.4222	0.2339	0.5494	0.2796
	MS	0.4133	0.0901	0.2582	0.0735	0.3924	0.0898	0.7085	0.1306
500	M	1.1044	1.833	0.9821	1.7663	1.095	1.8200	1.1351	1.835
	B	0.2971	0.1714	0.2140	0.1395	0.3283	0.1882	0.3679	0.2021
	MS	0.1743	0.0481	0.1099	0.0392	0.2137	0.0573	0.3059	0.0694
1000	M	1.0416	1.8113	0.9942	1.7867	1.0479	1.8125	1.0715	1.8214
	B	0.1963	0.1181	0.1223	0.0804	0.212	0.1260	0.2436	0.1408
	MS	0.0714	0.023	0.0494	0.0178	0.0856	0.0263	0.1151	0.0324

Table 6

M, B and MS of the STITLR Distribution $\alpha=0.5, \theta=3.0$

n	Properties	MLE		MPS		A.D		CVM	
		α	θ	α	θ	α	θ	α	θ
20	M	0.7220	3.0834	0.4508	2.5334	0.6005	2.9066	0.6586	3.0059
	B	0.4519	0.8929	0.3691	0.9934	0.3895	0.9073	0.418	0.9127
	MS	0.2969	1.0137	0.1852	1.2962	0.2146	1.0481	0.2563	1.0432
50	M	0.6473	3.0928	0.4596	2.6971	0.5925	2.9909	0.6349	3.0587
	B	0.3435	0.7021	0.2944	0.7437	0.3262	0.7129	0.3532	0.7443
	MS	0.1738	0.6582	0.1215	0.765	0.154	0.6674	0.1801	0.7178
70	M	0.6280	3.0936	0.4775	2.7821	0.5855	3.0071	0.6272	3.0742
	B	0.3069	0.6341	0.2641	0.638	0.2931	0.6285	0.3255	0.671
	MS	0.1436	0.5497	0.1027	0.5867	0.1298	0.5456	0.1582	0.602
100	M	0.6102	3.0903	0.4836	2.829	0.583	3.0362	0.6078	3.0628
	B	0.2667	0.5449	0.2344	0.5478	0.2647	0.5596	0.2931	0.6055
	MS	0.1174	0.4282	0.0859	0.4499	0.1097	0.4351	0.132	0.4998
150	M	0.5851	3.0704	0.4842	2.8745	0.56	3.018	0.6024	3.088
	B	0.2283	0.4770	0.193	0.4451	0.2239	0.4754	0.2598	0.5293
	MS	0.0870	0.3346	0.0618	0.3188	0.0837	0.331	0.1095	0.3987
250	M	0.5645	3.0653	0.4935	2.9247	0.5479	3.0302	0.5849	3.0894
	B	0.1785	0.3751	0.1527	0.3439	0.1781	0.3765	0.2112	0.4265

300	MS	0.0579	0.2201	0.0412	0.2045	0.0551	0.2174	0.0767	0.2727
	M	0.5625	3.0706	0.4948	2.939	0.5344	3.0084	0.5661	3.053
	B	0.1673	0.3481	0.1396	0.3089	0.1658	0.3527	0.1987	0.4047
500	MS	0.0508	0.1926	0.0358	0.1723	0.0485	0.1921	0.069	0.2476
	M	0.5363	3.0421	0.4886	2.9446	0.5297	3.0199	0.5404	3.0388
	B	0.1246	0.2684	0.0998	0.221	0.1344	0.2882	0.1431	0.2998
1000	MS	0.0271	0.1136	0.0194	0.0992	0.0317	0.1306	0.0377	0.1444
	M	0.5140	3.0124	0.4966	2.9793	0.5163	3.0149	0.5236	3.0258
	B	0.0846	0.1868	0.0612	0.129	0.0892	0.1949	0.0991	0.2121
	MS	0.0125	0.0567	0.0095	0.0467	0.0142	0.062	0.0172	0.0716

Tables 1, 2, 3, 4, 5, and 6 present the comprehensive outcomes of the simulation study. As the sample sizes increase, both the biases and MSs exhibit a notable decline, indicating a significant improvement in the precision of the parameter estimates. This trend underscores the convergence of the estimated values toward the true parameter values, demonstrating the effectiveness and reliability of the estimators employed. Furthermore, the consistent reduction in these metrics across varying sample sizes reinforces the robustness of the proposed STITLR distribution, suggesting that larger samples yield increasingly accurate and dependable parameter estimates.

REAL LIFE APPLICATION

In this section, we fit the STITLR model to a real-life dataset and compare its performance with other extensions of the Rayleigh distribution. The competing models include the Odd Fretchet Inverse R (OFIR) model by Elgarhy and Alrajhi (2019), the Lomax R (LR) model by Venegas et al. (2019), the Generalized R (GR) distribution by Kundu and Raqab (2005), and the Half Logistic Inverse R (HLIR) distribution by Almarashi et al. (2020). The goodness-of-fit of the new model and the competing models was evaluated using negative log-likelihood (-LL), AIC, BIC, CAIC, HQIC, the KS test, and associated p-values.

Table 7 summarises the results, and Figure 5 displays the fitted PDF, CDF, survival function, and PP plot of the STITLR model applied to the dataset.

The dataset includes survival times, measured in months, for 20 patients with acute myeloid leukaemia, as reported by Afify et al. (2014): 2.226, 2.113, 3.631, 2.473, 2.720, 2.050, 2.061, 3.915, 0.871, 1.548, 2.746, 1.972, 2.265, 1.200, 2.967, 2.808, 1.079, 2.353, 0.726, and 1.958.

Table 7

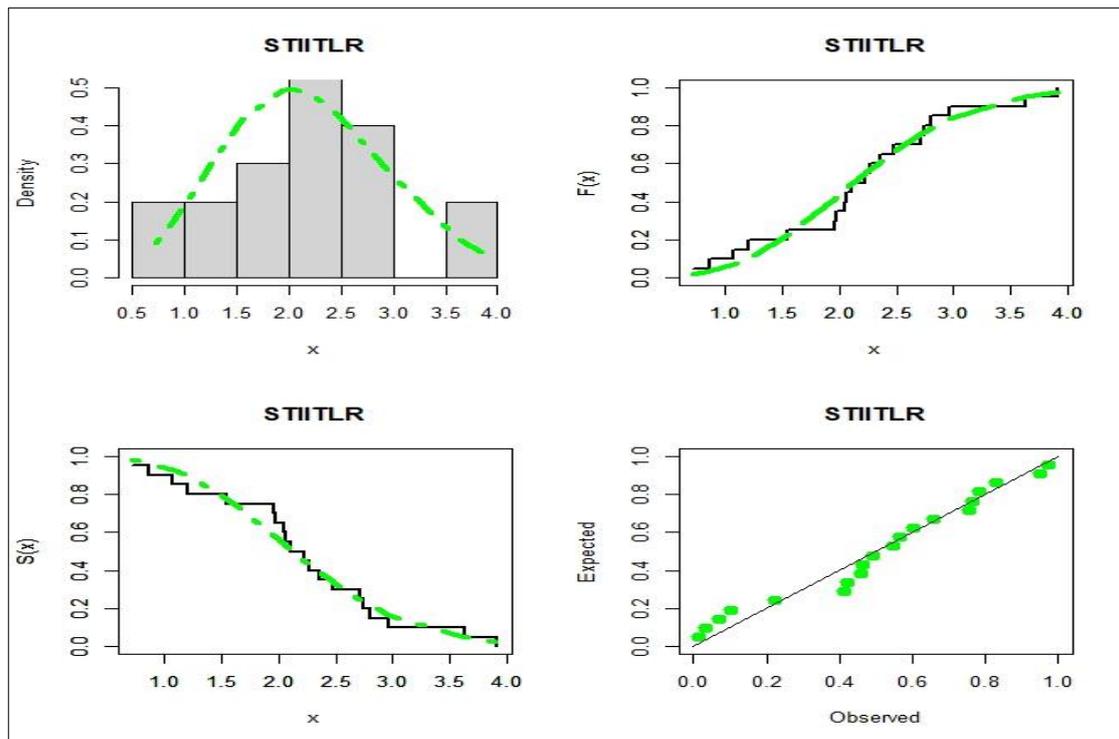
Goodness-of-Fit Test Results for the Dataset.

MODEL	MLE	-LL	AIC	BIC	CAIC	HQIC	KS	P value
STITLR	$\alpha = 0.4365$ $\theta = 1.2277$	24.3172	52.6343	54.6258	53.3402	53.0231	0.1656	0.5865
OFIR	$\alpha = 0.7166$ $\theta = 1.2677$	32.2106	68.4212	70.4126	69.1270	68.8099	0.3498	0.010
LR	$\alpha = 109.68$ $\theta = 593.97$	26.1192	56.2384	58.2298	56.9442	56.6271	0.2562	0.1205
GR	$\alpha = 1.8707$ $\lambda = 0.5146$	24.3689	52.7379	54.7293	53.4438	53.1266	0.1811	0.4733
HLIR	$\alpha = 1.6767$ $\beta = 1.7568$	27.2748	58.5497	60.5412	59.2556	58.9385	0.2689	0.0909

The results in Table 7 indicate that the STIITLR model outperforms the competing models, showing the lowest values for AIC, BIC, CAIC, HQIC, and KS. Furthermore, the KS test p-value supports the STIITLR model’s superiority, as it is the highest among all models, signifying a better fit to the data. This suggests that the STIITLR model provides a more suitable alternative for modelling this dataset. Given its strong fit, the STIITLR model could be a valuable tool in medical prognosis, particularly for estimating survival times and assessing mortality risk in patients with acute myeloid leukaemia. Such insights can aid healthcare providers in developing tailored treatment plans and providing precise survival estimates grounded in probabilistic modelling.

Figure 5

Fitted PDF, CDF, Survival Function and PP Plot of the STIITLR Model for the Dataset



As shown in Figure 5, the STIITLR model aligns closely with the empirical data, indicating a strong fit to the dataset. This alignment across the fitted PDF, CDF, survival function, and PP plot further supports the suitability of the STIITLR model for accurately modelling the observed survival times.

CONCLUSION

The advancement of lifetime modelling has prompted the development of more flexible distributions to accommodate the intricacies of empirical data. In response, this study proposed the Sine Type II Topp-Leone Rayleigh (STIITLR) distribution and explored its fundamental properties. Four different estimation methods were employed to estimate the model parameters, and a simulation study confirmed the consistency and reliability of the estimators. The practical utility of the proposed model was demonstrated through its application to biomedical survival data, where it outperformed several existing distributions. These results highlight the model’s strong potential to accurately capture the complex nature of survival data, offering valuable insights for biomedical and healthcare research. Looking ahead, future research may explore extensions of the STIITLR distribution by incorporating additional

parameters or alternative formulations to enhance its flexibility. Furthermore, evaluating the model's performance across a wider range of biomedical datasets would provide additional evidence of its robustness and support its broader adoption in clinical and epidemiological research.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

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