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PERFORMANCE MEASURE OF MULTIPLE-CHANNEL QUEUEING SYSTEMS WITH IMPRECISE DATA USING GRADED MEAN INTEGRATION FOR TRAPEZOIDAL AND HEXAGONAL FUZZY NUMBERS

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ABSTRACT

In this paper, a procedure to establish the different performance measures in terms of crisp value is proposed for two classes of arrivals and multiple channel queueing models, where both arrival and service rate are fuzzy numbers. The main idea is to convert the arrival rates and service rates under fuzzy queues into crisp queues by using graded mean integration approach, which can be represented as median rule number. Hence, we apply the crisp values obtained to establish the performance measure of conventional multiple queueing models. This procedure has shown its effectiveness when incorporated with many types of membership functions in solving queuing problems. Two numerical illustrations are presented to determine the validity of the procedure in this queueing model, which involved using trapezoidal and hexagonal fuzzy numbers. It can be concluded that graded mean integration approach is efficient with fuzzy queueing models to convert fuzzy queues into crisp queues. This finding has contributed to the body of knowledge by suggesting a new procedure of defuzzification as another efficient alternative.

Keywords: Multiple Channel Queueing Model, Two Class of Arrivals, Graded Mean Integration, Fuzzy Numbers, Performance Measures.

INTRODUCTION

Queueing problems are regularly expected in various areas of application such as manufacturing, industry, transportation, production systems and telecommunication. Queueing models play a significant role in decision making and design typically comprising a combination of decisions (Gross & Harris, 1985). This combination may involve various entities such as human and machine while operating certain processes e.g., the number of service counters at a facility, the effectiveness of arrival procedure especially in cases concerning priorities and the efficiency level of the machine. Many studies have been published in this research area where both rates, arrival and service are known (Derbala, 2005). However, there are cases where these parameters are unknown precisely. Study by van Vianen (2015) highlighted that the main obstacle in implementing evaluation of such queueing models is that it may be challenging to exactly estimate the real average waiting time of customers in the queue, particularly when it involves human factor. This factor has greater impacts to the scenario as humans are normally inconsistent. In some applications, the statistical information may be obtained subjectively; that is, the arrival and service pattern are more appropriately described by linguistic terms such as fast, moderate, or slow, rather than by probability distributions (Taha, 2003). The total time spent by a customer in the facility and on the queue can be described as having fuzzy elements due to some uncontrollable factors. Fuzzy input information of this kind will lessen the effectiveness of the quality of decisions if analyzed using conventional queueing decision models. Accordingly, fuzzy queueing decision problems deserve further investigation to find their appropriateness in measuring performance.

Many researchers such as Zadeh (1965) and Buckley (1990) proposed methods and techniques to merge between probability theory and possibilities, while other researchers proposed mathematical procedures; nonparametric linear programming technique with single priority queueing systems (Devaraj & Jayalakshmi, 2012a; Devaraj & Jayalakshmi, 2012b). On the other hand, some other studies used Yager's ranking method (Palpandi & Geetharamani, 2013) to convert fuzzy queues into crisp queues with the help of alpha cut while some others used graded mean approach (Ritha & Menon, 2012a; Robert & Ritha, 2010b; Mueen et al., 2017a). Most previous research focused on trapezoidal and triangular membership functions, where only a few studies adopted other types of linear membership functions, such as hexagonal fuzzy numbers (Mueen et al., 2017b). The usability of hexagonal membership function is seen in its vast application areas such as its recent application in transportation problems (Rajarajeshwari et al., 2013). In this paper, a new procedure is proposed to obtain the expected waiting time of customer in the queue for both classes using graded mean integration method with trapezoidal and hexagonal membership function for multiple channel models under two classes of queueing model. Then, the validity of this approach leads to the evaluation of the whole queueing system. The organization of this paper is as follows; the first section introduces the preliminaries of fuzzy set theory and membership functions regarding graded mean approach, while the next section explains the multiple channels under two classes of arrivals. The final sections demonstrate the numerical examples of the model proposed followed by discussion and conclusion.

CONCEPTS OF FUZZY SET THEORY

This section discusses some basic definitions and mathematical operations of fuzzy numbers are summarized which are quite vital for this article:

Definition 1: Fuzzy Sets

A fuzzy set is specified by membership function containing the components of a domain space or universe X in the interval [0, 1], that is $\tilde{A}=\left\{\left\{\left(x,\ \mu_{\tilde{A}}(x)\right)|x\epsilon X|\right\}\right\}$. Here $\mu_{\tilde{A}}: Z \rightarrow [0,1]$ is an interval called the degree of membership function of the fuzzy set, $\mu_{\tilde{A}}(z)$ represents the membership value of z∈Z in the fuzzy set Ã and the degree of membership is defined by R → [0,1].

Definition 2: Fuzzy Number

A fuzzy set Ã of a universe of discourse *X* is called a normal fuzzy set if there exists at least z∈Z such that $\mu_{\tilde{A}} = 1$. A fuzzy set Ã is convex if and only if for any z∈Z, the membership function of Ã satisfies the condition of $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$, where $0 \leq \lambda \leq 1$.

Definition 3: Trapezoidal Fuzzy Number

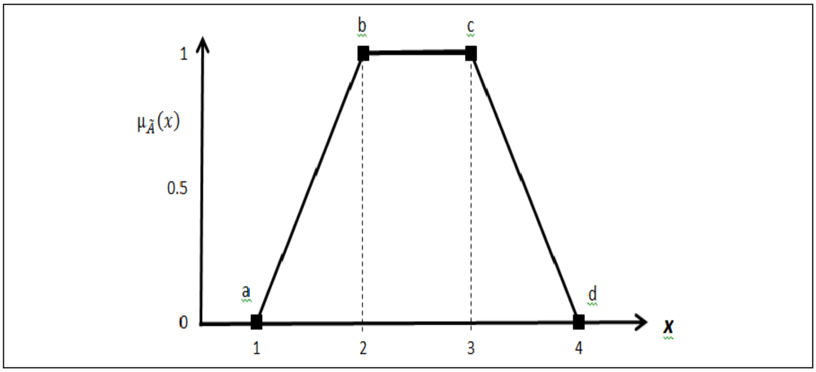
A trapezoidal fuzzy number Ã can be represented by Ã= (*a, b, c, d; 1*). Study by Banerjee and Roy (2012) adopted the most linear membership function defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & otherwise \end{cases} \quad (1)$$

where *a,b,c* and *d* represent the points inside closed interval.

Figure 1

Graphical Representation of Trapezoidal Fuzzy Number



Definition 4: Operations of Trapezoidal Fuzzy Number

Let $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. The arithmetic operations are defined as:

$\tilde{A} = (a_1, b_1, c_1, d_1), \tilde{B} = (a_2, b_2, c_2, d_2)$

- Addition: $\tilde{A} + \tilde{B} = [a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2, d_1 + d_2]$
- Subtraction: $\tilde{A} - \tilde{B} = [a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2]$
- Multiplication: $\tilde{A} * \tilde{B} = [a_1 . a_2, b_1 . b_2,\ c_1 . c_2, d_1 . d_2]$
- Division: $\frac{\tilde{A}}{\tilde{B}} = \left[\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}, \frac{d_1}{d_2}\right]$

Hexagonal Fuzzy Number

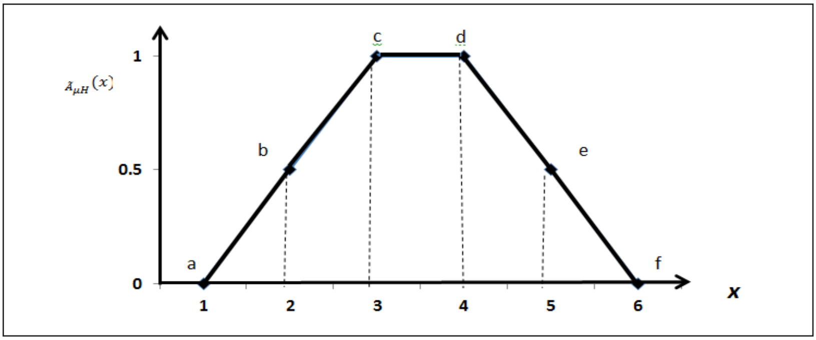
A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H = (a, b, c, d, e, f; 1)$, where a, b, c, d, f are real numbers (Rajarajeswari and Sangeetha, 2015). Its continuous membership function $\mu_{\tilde{A}_H}(x)$ is given by:

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0, & x < a \\ \frac{1}{2}\left(\frac{x-a}{b-a}\right), & a \le x \le b \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-b}{c-b}\right), & b \le x \le c \\ 1, & c \le x \le d \\ 1 - \frac{1}{2}\left(\frac{x-d}{e-d}\right), & d \le x \le e \\ \frac{1}{2}\left(\frac{f-x}{f-e}\right), & e \le x \le f \\ 0, & x > f \end{cases} \quad (2)$$

where *a,b,c,d,e* and *f* represent the points inside the interval.

Figure 2

Graphical Representation of Hexagonal Fuzzy Numbers



Operations of Hexagonal Fuzzy Numbers

Let $\tilde{A}_H = (a_1, b_1, c_1, d_1, e_1, f_1), \tilde{B}_H = (a_2, b_2, c_2, d_2, e_2, f_2)$ be two Hexagonal fuzzy numbers. The arithmetic operations are defined as:

- Addition: $\tilde{A}_H + \tilde{B}_H = [a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2, d_1 + d_2, e_1 + e_2, f_1 + f_2]$
- Subtraction: $\tilde{A}_H - \tilde{B}_H = [a_1 - a_2,\ b_1 - b_2,\ c_1 - c_2, d_1 - d_2, e_1 - e_2, f_1 - f_2]$
- Multiplication: $\tilde{A}_H * \tilde{B}_H = [a_1.a_2, b_1.b_2,\ c_1.c_2, d_1.d_2, e_1.e_2, f_1.f_2]$
- Division: $\frac{\tilde{A}_H}{\tilde{B}_H} = \left[\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}, \frac{d_1}{d_2}, \frac{e_1}{e_2}, \frac{f_1}{f_2}\right]$

Generalized Fuzzy Numbers

A generalized fuzzy number is a fuzzy set $\tilde{A}$ described in the universal set of real numbers represented R with membership function characteristics defined as follows:

$\mu_{\tilde{A}}: R \to [0, w]$ is a continuous mapping from R to the closed interval [0,1].
$\mu_{\tilde{A}}(z) = 0$ for all ; $z \in (-\infty, a] \cup [d, \infty)$;
$\mu_{\tilde{A}}(z)$ is strictly increasing on [a,b]; also, strictly decreasing on [b,d];
$\mu_{\tilde{A}}(z) = w$ for all $z \in [b, c]$; where $0<w\leq 1$.

Graded Mean Integration Method

Chen and Hsieh (1998) proposed graded mean integration approach for generalized fuzzy number. To describe this graded mean integration approach, suppose L^{-1} are R^{-1} inverse function left and right respectively and the graded mean h-level value of generalized fuzzy number of trapezoidal defined by:

$$P(\tilde{A}) = \int_0^w h\left(\frac{L^{-1}(x) + R^{-1}(x)}{2}\right) dh \Big/ \int_0^w h\, dh, \tag{3}$$

where h between 0 and w, $0<w\leq 1$.

$$L^{-1}(x) = a + \left(\frac{b-a}{w}\right)z \tag{4}$$

$$R^{-1}(z) = b + \left(\frac{c-b}{w}\right)z \tag{5}$$

By equation (3), the graded mean integration representation of A is:

$$P(\tilde{A}) = \frac{w(a + 2b + 2c + d)}{6}. \tag{6}$$

Similarly, the hexagonal fuzzy numbers $\tilde{A}(z) = (a, b, c, d, e, f; w)$

where

$$L^{-1}(z) = \frac{1}{2}\left(\frac{x-a}{b-a}\right) + \frac{1}{2} + \frac{1}{2}\left(\frac{x-b}{c-b}\right)z, \tag{7}$$

and

$$R^{-1}(z) = 1 - \frac{1}{2}\left(\frac{x-d}{e-d}\right) + \frac{1}{2}\left(\frac{f-x}{f-e}\right)z. \tag{8}$$

By equation (3), the graded mean integration of hexagonal fuzzy numbers is:

$$P(\tilde{A}) = \frac{w(2a + 3b + 4c + 4d + 3e + 2f)}{18}. \tag{9}$$

Fuzzy Multiple Channel with Two Class of Arrivals

Consider a multiple channel queueing model with two types of arrival class. In more detail, this is a queueing system in which arrivals take place in multiple channels with two classes of non-preemptive priorities according to Poisson streams. The arrival rate of the first class is denoted by $\widetilde{\lambda_1}$ and $\widetilde{\lambda_2}$ is the denotation for the second class with the fuzzy service rates of all customers following an exponential distribution having same service time with fuzzy rate denoted by $\tilde{\mu}$. Customers are served as two classes of priority and the corresponding model is denoted by *(FM1,FM2)/FM/C/PR/∞/∞*, where *PR* represents priority with size of system and population being infinity. In this model, the arrival rates and service rates will be represented as fuzzy sets defined:

$$\widetilde{\lambda_1} = \{(w, \mu_{\widetilde{\lambda_1}}(w))/w \in W\}, \tag{10}$$

$$\widetilde{\lambda_2} = \{(x, \mu_{\widetilde{\lambda_2}}(x))/x \in X\}, \tag{11}$$

$$\tilde{\mu} = \{(y, \mu_{\tilde{\mu}}(y))/y \in Y\}. \tag{12}$$

where *W, X* and *Y* are crisp universal sets of the corresponding arrival and service rates. Setting *p(w,x,y)* to denote the system characteristic of interest, $p_{(\widetilde{\lambda_1},\widetilde{\lambda_2},\tilde{g})}$ is a fuzzy number since , $\widetilde{\lambda_1}, \lambda_2$ and $\tilde{g}$ are fuzzy numbers. From previous knowledge and results in Adan et al. (2001) and Chen and Chien-Chung (2006), we have the expected waiting time of customer in the crisp priority queuing model with two classes, (M1,M2)/M/C/PR and steady state $\rho = \lambda_1 + \lambda_2/c\mu < 1$ obtained as:

$$W_q^{(1)} = \frac{\prod w}{(1-\rho_1)} \cdot \frac{E[R]}{c}, \tag{13}$$

and

$$W_q^{(2)} = \frac{\prod w}{(1-\rho)(1-\rho_1)} \cdot \frac{E[R]}{c}, \tag{14}$$

Where

$$\rho = \rho_1 + \rho_2 , \tag{15}$$

$$\prod w = \frac{(c\rho)^c}{c!}(1-\rho)\left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!\,(1-\rho)}\right]^{-1} ;n = 0,1,2,\ldots \tag{16}$$

and

$$E[R] = \frac{E[B^2]}{2\mathrm{E[B]}}. \tag{17}$$

In this existing queueing model, $\prod w$ represents the probability of customer waiting in the queue and E[R] represents the residual processing time which is needed to complete the service. The other performance measurements are defined by:

$$L_q^{(i)} = \lambda_i W_s^{(i)}, \tag{18}$$

$$W_s^{(i)} = W_q^{(i)} + \frac{1}{\mu}, \tag{19}$$

$$L_s^{(i)} = \lambda_i W_s^{(i)}, \tag{20}$$

where i=1,2.

NUMERICAL ILLUSTRATION

Consider a line production of machine receiving two types of Poisson arrival rates, $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ with service time represented as an exponential distribution with rate, $\tilde{\mu}$ and the number of channels in the system being two. Note that all values are under fuzzy environment with corresponding model *(FM1,FM2)/FM/2/PR/∞/∞*. The management wants to compute the expected waiting time of customers in each class, hence evaluating the entire system. We start from the defuzzification stage to obtain crisp values for average arrival rates for class one and two together with the average service time in the queueing system from trapezoidal fuzzy number and hexagonal fuzzy number. Then the next stage is to compute the performance measures.

The Trapezoidal Membership Function

Assume that arrival rate and service rates for two classes are trapezoidal fuzzy numbers, and they are defined as:

$$\tilde{\lambda}_1 = [1,2,3,4], \tilde{\lambda}_2 = [5,6,7,8],$$

and

$\tilde{\mu} = [\,8,9,11,12]$.

From equation (4), the ranking of $\tilde{\lambda}_1$, $\tilde{\lambda}_2$ and $\tilde{\mu}$ are obtained as follows:

$$R(\tilde{\lambda}_1) = R(1,2,3,4;1) = \frac{(1+4+6+4)}{6} = 2.5, \quad (21)$$

$$R(\tilde{\lambda}_2) = R(5,6,7,8;1) = \frac{(5+12+14+8)}{6} = 6.5, \quad (22)$$

$$R(\tilde{\mu}) = R(8,9,11,12;1) = \frac{(8+18+22+12)}{6} = 10. \quad (23)$$

The Hexagonal Membership Function

Assume that both arrival and service rates for two classes are hexagonal fuzzy numbers, and they are defined as:

$$\tilde{\lambda}_1 = [1,2,3,4,5,6], \tilde{\lambda}_2 = [7,8,9,10,11,12],$$

and

$$\tilde{\mu} = [13,14,15,16,17,18].$$

According to equation (5), the ranking values of $\tilde{\lambda}_1, \tilde{\lambda}_2$ and $\tilde{\mu}$ are obtained as:

$R(\tilde{\lambda}_1) = 3.5, R(\tilde{\lambda}_2) = 9.5$ and $R(\tilde{\mu}) = 15.5$.

On the other hand, from observation, the service time of customer will be random variable. Hence, the mean residual processing time by the first two moments of the processing time is calculated as general distribution. Therefore, the mean residual time in this model is assumed to follow exponential distribution, given by $E[B] = 1/\mu$ and $E[B^2] = 2/\mu^2$. Then $E[R] = E[B]$. By substituting the value of *E[R]* into performance measurements represented by $W_q^{(1)}$ and $W_q^{(2)}$ respectively.

With reference to Equations (14)-(16) for calculating the performance measures which translates to evaluating the system, we obtain the results as seen in table 1. This displays the different performance measure values for each priority class when considering the two types of membership functions under consideration.

Table 1

Different Performance Measurements of Two Membership Functions

MF.	$W_q^{(1)}$	$W_q^{(2)}$	$L_q^{(1)}$	$L_q^{(2)}$	$W_s^{(1)}$	$W_s^{(2)}$	$L_s^{(1)}$	$L_s^{(2)}$
Tp.	0.034	0.062	0.085	0.403	0.134	0.162	0.335	1. 053
Hex.	0.014	0.026	0.051	0.254	0.079	0.091	0.277	0.867

DISCUSSION AND ANALYSIS

The following observations can be drawn from table 1 as follows:

1. It is essential to ensure the establishment of the arrival rate or service rate are not vague so that the performance measures obtained are precise. By applying graded mean approach with two types of membership functions such as trapezoidal and hexagonal fuzzy numbers, we convert fuzzy queues into crisp queues and obtain real value.
2. Fuzzy set theory is a powerful tool in assisting the decision maker to convert and remove the ambiguity from data and become more realistic to solve complex problems in queueing system.
3. Generally, we notice the values of performance measurements in hexagonal membership functions are less than the values of

trapezoidal membership functions, this leads to extending the area of fuzzy numbers and the results becomes better.

4. All values of performance measurements of class one is less than class two in this type of model.

CONCLUSION

In this paper we conclude that graded mean integration approach is efficient with fuzzy queueing models to convert fuzzy queues into crisp queues. The novel approach of using symmetrical hexagonal fuzzy numbers with this method give us superior results under performance measures. Hence, this leads to extend more area to the decision maker to obtain different values and more information. Therefore, it is convenient way to evaluate systems. For future work this approach can be used with other piecewise linear membership functions such as diagonal, octagonal membership functions to evaluate queueing systems.

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